THE
BAKHSHALI MANUSCRIPT
AN ANCIENT TREATISE OF INDIAN ARITHMETIC

EDITED BY

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and
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ALLAHABAD.
Dr. Ratna Kumari

Born 20-3-1942  
Died 2-12-1964
Dr. Ratna Kumari, M. A., D. Phil. was deeply interested in education, higher research and scholarship, and when she died in 1964, the Director of the Research Institute of Ancient Scientific Studies, New Delhi, graciously agreed to publish in her commemoration a Series to be known as the "Dr. Ratna Kumari Publication Series", and under this arrangement, the five volumes published were: Satapatha Brahmanam. Vol. I, II and III (1967, 1969, 1970); Hudhayana Sulba Sutram (1968) and the Apastamba Sulba Sutrum (1968). It is to be regretted that in 1971, Pundit Ram Swarup Sharma, the Director of the Institute died and shortly afterwards, the activities of the Institute came to a close. In 1971, from an endowment created by the relations of late Dr. Ratna Kumari, Dr. Ratna Kumari Svadhyaya Sansthana, a research organisation for promotion of higher studies amongst ladies, was established at Allahabad, with Sri Anand Prakash, the younger son of Dr. Ratna Kumari as the first President. Svami Satya Prakash (formerly, Prof. Dr. Satya Prakash) has authorised Dr. Ratna Kumari Svadhyaya Sansthana to publish several of his works, particularly, all of them which were published by the Research Institute of Ancient Scientific Studies, West Patel Nagar, New Delhi, and has assigned the right of publication of these works to the Sansthana.

We are pleased to offer to the public for the first time, the classified text of the Bakhshall Manuscript in Devanagari script. The manuscript was literally dug out of a mound at Bakhshuli on the north-west frontier of India in 1881, and Dr. Hoernle carefully examined the manuscript and published an English translation of a few of the leaves in 1888. This being the earliest manuscript on Indian Arithmetic, it aroused considerable interest in the West. The Archaeological Survey of India published the manuscript in Parts I, II and III, as edited by G. R. Kaye. In 1902, Dr. Hoernle presented the manuscript to the Bodleian Library. This distinguished work of Kaye is now not available in market. Dr. Bibhuti Bhusan Datta of Calcutta University also made detailed study of this manuscript.
Dr. J. Jyotishmati, a distinguished member of the Sansthan, took considerable pains in editing the text, and we are obliged to Svami Saty Prakash Sarasvati for his supervision and critical introduction. We are also obliged to Sri Jagdish Prasad Misra, B A., LL. B., for his dedicated services to the Sansthan, particularly in the publication programmes.

We regret to announce the premature death of our first President Sri Anand Prakash on December 13, 1976. He was a distinguished graduate of the University of Allahabad, and took his graduation in Mechanical Engineering from the University of Glasgow, and since then he had been working in industry. To cherish his memory, we have the privilege of dedicating this little volume.

The Sansthan
Allahabad
Ramanavami
April 5, 1979.

Prabha Grover
M Sc., D. Phil.
Director
Anand Prakash

Born 4-4-1938    Died 13-12-1976
To Our President

ANAND PRAKASH

who died at

an early age of 38.
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INTRODUCTION

Chajakputra—The Scribe:

While we pay homage to the unknown author of a text which came to be known as the Bakhshali Manuscript, we should express our gratitude to another person of unknown name who scribed the manuscript from a text quite lost to us, and not only that, since this scribe was himself a mathematician of no less repute, being known as a prince amongst calculators, he added illustrations and often added something to the running commentary of this Text. Though we have no independent source to corroborate, it is definite that a Text existed as early as the second or third century A.D., if not earlier; this Text was traditionally used by the lovers of calculations (teachers and taughts) in some parts of the country (especially the north-western parts of the Aryavartta), and as it was handed down to us generation after generation, new worked and unworked problems were added to the existing material. At the end of the manuscript (or somewhere on Folio 50, recto) we find a colophon mentioning that the work was scribed by a certain Brahmana, a prince of calculators the son of Chajaka (চাহজ কা). Thus we know the name of the father of the scribe only. I would like to call the scribe of the present manuscript himself as Chajaka-Putra (চাহজক পুত্র), the son of Chajaka. He himself is not an author of the Text; he merely copies it out from some other text already current. Even if the date of copying this text were so late as the tenth or eleventh century and even if some illustrations included in the manuscript were not of a very early origin, the Bakhshali Text has its great importance, this being one of the earliest Texts in history available to us on the science of calculations.

How the Text was Found?

In 1881, a mathematical work written on birchbark was found at Bakhshali near Mardan on the north-west frontier of India. This manuscript was supposed to be of great age and its discovery aroused considerable interest. Part of it was examined by Dr. Hoernle,
who published a short account of it in 1883\textsuperscript{1}, and a fuller account in 1886\textsuperscript{2}, which together with the translation of a few of the leaves was republished in 1888\textsuperscript{3}. Dr. Hoernle had intended, in due course, to publish a complete edition of the Text, but was unable to do so. The work was later on printed and published in 1927, under the title "The Bakhshali Manuscript", Part I and II, by the Government of India with photographic fascimiles and transliteration of the Text together with a very comprehensive introduction by G. R. Kaye\textsuperscript{4}. This was followed by the publication of the Bakhshali Manuscript, Part III, in 1933 as the Text Rearranged shortly after Kaye's death. Dr. Bibhutibhushan Datta, a distinguished worker in the field of Indian mathematics, also published a critical review on the Bakhshali Manuscript\textsuperscript{5}.

Bakhshali (or Bakhshalai, as it was written in the official maps) is a village of the Yusufzai subdivision of the district of Peshawar of the North-Western Frontier of India (now in Pakistan). It is situated on, or near, the river Mukham, which eventually joins the Kabul river near Nowshers, some twenty miles further south. Six miles W. N. W. of Bakhshali is Jamalgarhi, twelve miles to the west of Takht-i-Bhai and twenty miles W. S. W. is Charsada, famous for their Indo-Greek art treasures.

Bakhshali is about 150 miles from Kabul, 160 from Srinagar, 50 from Peshawar, 350 from Bakh and 70 from Taxila. It is in the Trans-Indus country and in ancient times was within Persian boundaries—in the Arachosian satrapy of the Achaemenid kings. It is within that part of the country to which the name Gandhara

1. Indian Antiquary, XII (1883), pp. 89—90.
2. Verhandtungen des VII Internationalen Orientalisten Congresses, Arische Section, pp. 127 et. seq.
has been given, and was subject to those western influences which are so bountifully illustrated in the so-called Gandhara art.

The authentic record of the discovery of the manuscript appears to be contained in the following letter dated the 5th of July, 1881, from the Assistant Commissioner at Mardan.

"In reply to your No. 1306, dated 20th ultimo, and its enclosures, I have the honour to inform you that the remains of the papyrus MS. referred to were brought to me by the Inspector of police, Mian An-Wan-Udin. The finder, a tenant of the latter, said he had found the manuscript while digging in a ruined stone enclosure on one of the mounds near Bakhshali. These mounds lie on the west side of the Mardan and Bakhshali roads and are evidently the remains of a former village. Close to the same spot the man found a triangular-shaped 'diva', a soap-stone pencil, and a large lota of baked clay with a perforated bottom. I had a further search made but nothing else was found.

"According to the finder's statement the greater part of the manuscript had been destroyed in taking it up from the place where it lay between stones. The remains when brought to me were like dry tinder, and there may be about fifty pages left some of which would be certainly legible to any one who knew the characters. The letters on some of the pages are very clear and look like some kind of Prakrita, but it is most difficult to separate the pages without injuring them. I had intended to forward the manuscript to the Lahore Museum in the hope that it might be sent on thence to some scholar, but I was unable to have a proper tin box made for it before I left Mardan. I will see to this on my return from leave. The papyrus will require very tender manipulation. The result will be interesting if it enables us to judge the age of the rules where the manuscript was found."

1. Apparently the manuscript was found in May 1881.
2. General Cunningham in a private letter to Dr. Hoernle, dated Simla, 5th June, 1882, says: "Bakhshali is 4 miles north of Shahbazgarh. It is a mound with the village on the top of it. The birch-bark manuscript was found in a field near a well without trace of any building near the spot, which is outside the mound village....".
3. Kaye has, however, expressed doubts regarding the authenticity of this account, for, he thinks, it was written apparently from memory, a month or so after the discovery of the manuscript.
In the meantime notices of the discovery had found their way into the Indian newspapers. Professor Buhler, who had read of the discovery in the "Bombay Gazette,¹ communicated the announcement to Professor Weber, who brought it to the notice of the Fifth International Congress of Orientalists then assembled in Berlin. In Professor Buhler's letter to Prof. Weber it was stated that the manuscript had been found "carefully enclosed in a stone chamber" and it was thought that the newly discovered manuscript might prove to be "one of the Tripitakas which Kaniska ordered to be deposited in Stupas".

Kaye, however, says that there is nothing whatever in the record of the find to justify Buhler's statement, which seems to have originated in a rather strange interpretation of the words "while digging in a stone enclosure" that occur in the letter quoted above, and which are themselves of doubtful reliability.

The manuscript was subsequently sent to the Lieutenant-Governor of the Punjab, who, on the advice of General Cunningham, directed it to be transmitted to Dr. Hoernle, then head of the Calcutta Madrasa, for examination and publication. In 1882, Dr. Hoernle gave a short description of the manuscript before the Asiatic Society of Bengal, and this description was published in the Indian Antiquary of 1883. At the Seventh Oriental Conference, held at Vienna in 1886, he gave a fuller account which was published in the proceedings of the Conference, and also with some additions, in the

¹. This account appears in the Bombay Gazette of Wednesday, August 13th, 1881 and is as follows:

"The remains of a very ancient papyrus manuscript have been found near Bakhshali, in the Mardan Tahsil, Peshawar District. On the west side of the Mardan and Bakhshali road are some mounds, believed to be the remains of a former village, though nothing is known with any certainty regarding them, and it was while digging in a ruined stone enclosure on one of these mounds the discovery was made. A triangular-shaped stone "Diwa", and a soap-stone pencil and a large lotah of baked clay, with a perforated bottom, were found at the same place. Much of the manuscript was destroyed by the ignorant finder in taking it up from the spot where it lay between the stones; and the remains are described as being like dry tinder, in some of the pages. However, the character, which somewhat resembles 'prakrita' (प्राकृत), is clear, and it is hoped it may be deciphered when it reaches Lahore, whither we understand it is shortly to be sent."
The Manuscript

The manuscript consists of some 70 leaves of birchbark, but some of these are mere scraps. The largest leaf measures about 5.75 by 3.5 inches or 14.5 by 8.9 centimetres. The leaves, which are numbered according to the Bodleian Library arrangement from 1 to 70, may be classified according to their size and condition as follows:

- In fair condition but broken at the edges—size, not less than 5 by 3 inches (13 by 8 cms.)
  1, 2; 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68—Total 35.

- Rather more damaged but otherwise in fair condition not less than 4 by 2 inches (12 by 5 cms.)
  2, 23, 26, 42, 45, 46, 48, 50, 51, 52, 55, 56, 57, 58, 64, 69—Total 16.

- Much damaged

Scrap.

27, 28, 29, 33, 35, 38, 39, 40, 42, 54, 68, 70—Total 11.

One folio (19) is entirely blank.

Certain folios consist of two leaves stuck together, namely 7, 31 and 65, and possibly others. It would not be difficult to separate these double leaves without damaging the manuscript.

The leaves are now mounted between sheets of mica and placed within an album. The mica sheets are about 7.4 by 4.6 inches and are fixed together by strips of gummed paper at the edges leaving a clear area of 61 by 34 inches.
Possibly the original strip of birch-bark from which the leaves of Bakhshali manuscript were taken was roughly of the shape of the annexed diagram and was cut up into the oblongs indicated. If A, B, C, etc., represent the upper layer, and A', B', C', etc. the lower layer, then, according to the evidence of the leaves themselves, they were arranged for purposes of writing upon in the order A, A'; B, B'; C, C'; etc., or A, A'; D, D'; etc.

Regarding the format of the Bakhshali manuscript (ratio 1.7) as a criterion of age, Kaye says that he could come to no positive conclusion from the evidence before him. Dr. Hoernle, however, writes as follows:

"It is noteworthy that the two oldest (Indian paper) manuscript known to us point to their having been made in imitation of such a birch bark prototype as the Bakhshali manuscript."

Kaye, however, does not accept this argument, for it would be quite as reasonable to conclude that the Bakhshali format was determined by the paper manuscript formats, and that it is of later date than the introduction into India of paper as a writing material; and this, according to Kaye, would place the Bakhshali manuscript about the twelfth century of our era at the earliest.

The Script

The Bakhshali text is written in the Sarada script, which flourished on the north-west borders of India from about the Ninth Century until within recent times. Its distribution in space is fairly definitely limited to a comparatively small area lying between longitudes 72 and 88 east of Greenwich and latitudes 32 and 36 north. Dr. Vogel distinguishes between Sarada proper, of which the latest examples are of the early Thirteenth Century, and modern Sarada.

The writing of the Bakhshali manuscript is of the earlier period and is generally very good writing indeed. It was written by at least
Old Indian Scripts

SH = Sharda (Modern) ; NA = Nagarî
BR = Brahmi ; BO = Bower MS ; AC = Acute-angled ; SA = Saraband ; BA = Bakshali MS ; BAJ = Bajjapal
A page of the Bakhshali Manuscript
(17 verso)
two scribes. Kaye gives a detailed account of it, and has classified the writing styles also (α and β as he denotes them). Style α is divided into four sub-sections which possibly belong to the work of four separate scribes, although it is not easy to point out any fundamental differences between these styles. Folio 65 possibly exhibits the writings of two separate scribes on the two sides, which do not belong to the same original leaf.

The style β is distinguished by its boldness by 'tails' or flourishes (including very long virama marks), the methods of writing mediale, ai and o, the looped six, etc., etc.

A particular section, which Kaye calls as M, has no example of ai and contains practically all the examples of "o". Sections L (α₄) and G (α₃) show marked differences in the methods of writing mediale, e, while section F is, in this matter, very much like section L. Section C (α₂) has no example of either the JIHVAMULIYA (जिघवमलःलः) or UPADHMANIYA, (उपधमानीय), and so on; but it must be borne in mind that these statistics are only of value in the mass.

The Language

The language of the text may be described as an irregular Sanskrit. Nearly all the words used are Sanskrit, and the rules of Sanskrit grammar and prosody are followed with some laxity. The peculiarities of spelling, sandhi, grammar, etc., that occur in the text are exceedingly common in the inscriptions of the Eleventh and Twelfth Centuries found in the north-west of India. Dr. Hoernle, however, implies that the language is much older. He states that the text "is written in the so-called Gatha dialect, or in that literary form the North-Western Prakrit which which preceded the employment, in secular composition, of the classical Sanskrit". He also states that this dialect "appears to have been in general use, in North-Western India, for literary purposes, till about the end of the Third Century A. D".

Order and Arrangement of Folios

The Bodleian Library order of the folios, which was definitely fixed by circumstances not within the control of the editor, has
necessarily been followed by Kaye in his edition, in the arrangement of the facsimiles and the first transliteration of the text (Part I and Part II). The whole material was rearranged in the posthumous publication: *The Bakhshali Manuscript*, Part III.

The leaves were disarranged to some extent before they reached Dr. Hoernle, but unfortunately he did not leave any proper record of the order in which the leaves reached him.

Dr. Hoernle attempted to arrange the leaves on the basis of the numbered sutras; but the numbered sutras were too few and too unevenly distributed to serve this purpose, and in some ways they were even misleading. The chief criterion of order is, of course, the nature of the contents of the leaves, but Kaye made an attempt to utilise all available criteria as help towards the solution of the problem of order. Had all the leaves been extant, even if fragments, the problem could have been completely solved; but some leaves are completely missing and many fragments have disappeared altogether, so that the problem could be only partially soluble. Sometimes, even the knots in the birch-bark were of assistance, and besides this natural aid there was the accidental one of the effects of the method of storage. Possibly for some hundreds of years the bundle of leaves was subject to a certain amount of pressure, and was exposed particularly at the edges, to chemical and other disintegrating actions. Some of the leaves stuck together, the edges of all became frayed and certain leaves became so frail as to break up into scraps when handled. On the principle that contiguous leaves would be affected approximately to the same extent, we might, if no disintegrating effects had taken place since the find, rebuild in layers the original bundle. But we know that further disintegration has taken place; however, similarity in size and shape and mechanical makings were of distinct help in rearranging the leaves.

**Contents of the Manuscript**

The portions of the manuscript that have been preserved are wholly concerned with mathematics. Dr. Hoernle described the work in 1888 in the following words:—

"The beginning and end of the manuscript being lost, both the name of the work and its author are unknown. The subject of the
work, however, is arithmetic. It contains a great variety of problems relating to daily life. The following are examples:

(i) ‘In a carriage, instead of 10 horses, there are yoked 4, the distance traversed by the former was one hundred; how much will the other horses be able to accomplish?'

(ii) The following is more complicated: ‘A certain person travels 5 yojanas on the first day, and 3 more on each succeeding day; another who travels 7 yojanas on each day, has a start of 5 days; in what time will they meet?’

(iii) The following is still more complicated: ‘Of 3 merchants, the first possesses 7 horses, the second 9 ponies, the third 10 camels; each of them gives away 3 animals to be equally distributed among themselves. The result is that the value of their respective properties become equal: how much was the value of each animal?’

“The method prescribed in the rules for the solution of these problems is extremely mechanical and reduces the labour of thinking to a minimum”.

The following is a summary list of the contents of the work as far as its present state allows of such analysis:

| Section |
|-----------------|-----------------|
| Problems involving systems of linear equations | A |
| Indeterminate equations of the second degree | A and K |
| Arithmetical progressions | B and C |
| Quadratic equations | C |
| Approximate evaluations of square-roots | C |
| Complex series | F |
| Problems of the type $x \prod (1 - a_n) = P$ | G |

1. नियोरथोश्वैदंशिवाभावते हृय पद्वकम्। गन्तव्यं योजनशतम्...
   किमुद्भवेत्। इ १० ह्य लगभगस्य ५। गन्तव्ययोजन १००।
   —Folio 8, verso.

2. आदि पद्वकम् उत्तरं कीर्ण नरो योजन गम्यते। विभिन्न प्रतिवर्त्तत्वत्तमार्थ बिन पद्वकम्। केन कालेन समता कथ्यता गणकोलम।
   —Folio 6, recto

3. ...विभिन्नकर्तव्युण्डः नवः। कपड़ा बन शून्ययथा।...वर्त्त च परस्परे पृथक्कल्प धर्मं हुं विनावे मूल्यं वा प्राप्तिः पृथक्कवर्त्तवदि....वर्त्तं ततो मेतिकाविं मंगल:।
   —Folio 3, verso.
The computation of the fineness of gold
Problems on income and expenditure, and profit and loss
Miscellaneous problems
Mensuration

Such is a very rough outline of the work as it now stands. Perhaps the most interesting sections are C, A and M; and of these C is the most complete and was evidently treated as of considerable importance. Section A is also of special interest as it contains examples which may be described as of the epanthem type. Section M is of interest principally on account of the methods of expressing the numerous measures involved and also because of its literary and social references.

Generalized Arithmetic and Algebra

Although the work is arithmetical in form it would be misleading to describe it as a simple arithmetical text-book No algebraical symbolism is employed, but the solutions are often given in such a general form as to imply the complete general solution, i.e., the solutions, though arithmetical in form, are really generalized arithmetic, or algebra. First of all a particular rule is given, which is intended to apply to the particular set of examples that follows. These rules are often expressed in language that would be impossible to interpret without the light thrown upon them by the solutions. The examples are themselves sometimes trivial, but the solutions, often expressed with what at first glance appears to be meticulous care, often redeem the examples from their apparent triviality. Proofs or verifications are often given with some elaboration and on occasions are multiplied.

Kaye has said that the work may be divided roughly into algebraic, arithmetical and geometrical sections, knowing of course that the boundaries of these sections are not clear, but according to him it would be more correct to classify the problems as (a) academic, (b) commercial, and (c) miscellaneous.

Judging by the manuscript as it now stands, the problems involving geometric notions were comparatively very few, and we can only guess at the meanings of the remaining fragments dealing with this branch of mathematics.
Generalized Arithmetic and Algebra

One pleasing feature is the small space occupied by financial problems. There is only one problem on interest, and an unobtrusive section containing problems on profit and loss.

Those problems classed as academic are concerned with particular mathematical notions that in early mediavel times had a traditional value and interest, such as the *epanthema*, the *regula virgillum*, certain indeterminate equations of the second degree, and certain sets of linear equations.

The miscellaneous problems include examples where the interest is rather in the illustrative material than in the mathematical notions involved; e.g., there are problems concerned with the abduction of Sita by Ravana, the prowess of Haihaya, the constitution of an army, the Sun's chariot, the daily journey of the planet Saturn, gifts to Siva, etc. etc.

Such, or similar features, are, however, common to many mediavel mathematical works. The Bakhshali manuscript is, however, almost unique in at least two respects of some mathematical importance: (i) The first of these is the employment of a special sign in the form of a cross—exactly like our *plus* sign but placed after the quantity it effects—to indicate a negative quantity. (ii) The second special characteristic consists of the set of methods for indicating the change-ratios of certain measures.

Text is of Indian form

Whether of a purely Indian origin or not, Kaye says that the work is Indian in form. It is written in a sort of Sanskrit and generally conforms to the Indian text-book fashion but there are certain apparent omissions. Perhaps the most noteworthy feature of the classical Hindu texts is their treatment of indeterminate equations of the first degree, while their greatest achievement is the full solution of the so-called Pellian equation. A great part of the texts of Brahmagupta, Mahavira and Bhaskara are devoted to one or both of these topics, but there is no evidence of either in what remains of the Bakhshali text; and this apparent omission is the more noticeable, inasmuch as there is evidence of considerable skill in the treatment of systems of linear equations and certain indeterminates of the second degree. Another omission to note is of a different character altogether. Every early Hindu work of this kind has a section relating to
the "shadow of a gnomon", but in our text there is no evidence of
such a section. We must not, however, as Kaye says, "pay too
much attention to these apparent omissions. The possibility of entire
sections of the manuscript being destroyed is not great, but negative
evidence and a mutilated manuscript do not carry us very far.

Is the Work Homogeneous?

On the first examination of the manuscript it would be noticed
that the writing was not uniform, and that, in particular, certain leaves
differentiated themselves from the rest by a bolder and, on the whole,
a better style of writing; and Kaye has distinguished this set of leaves
as the "M" section. This early differentiation was a most useful one
for it marked not only a difference in style of writing but also one of
matter. Indeed this "M" section proved to have so many peculi-
arities that the idea that it was possibly a separate work could not be
ignored. But the rest of the manuscript is by no means uniform in
style of writing or anything else, and Kaye is not convinced that the
"M" section is the work of a separate author, although, he rather
suspects that it is. It is, however, pretty certain that it was the work
of a separate scribe; but, as there are slight indications that the
other portions of the manuscript were dictated, this does not affect
the question of authorship conclusively.

The peculiarities of the "M" section, although they may not
prove heterogeneity of workmanship, call for some special mention
and are here summarized.

1. The Script:

(a) The writing is bolder and, on the whole, more uniform
than that of the rest of manuscript.

(b) Flourishes or extensions of the bottom endstrokes are
common. These flourishes occur particularly at the end of ligatures,
but also in the cases of the numeral figures "5", "7" and "9", and
even in the case of the stop bars, and occasionally they even occur at
the ends of the frame-works of the "cells" (e. g., see fol. 47, etc.).

(c) The numerical symbols of section "M" have been shown
in a table by Kaye [not reproduced here, Table IV (7) line 1], where
the looped "6" should be noted. This is useful but not an infalli-
ble criterion,
Is the Work Homogeneous?

(d) The following table relating to the formation of medial vowels a, i, and o taken from Part II.

<table>
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<tr>
<th></th>
<th>ai</th>
<th>o</th>
<th>ai</th>
<th>o</th>
<th>o</th>
</tr>
</thead>
<tbody>
<tr>
<td>'M' section</td>
<td>0%</td>
<td>100%</td>
<td>32%</td>
<td>32%</td>
<td>36%</td>
</tr>
<tr>
<td>Whole, manuscript</td>
<td>24%</td>
<td>76%</td>
<td>75%</td>
<td>15%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Here the indication appears to be very definite indeed, but it must be borne in mind that such criteria only apply in the manuscript, that the total number of ai examples is only 19, and so on.

II. It is curious that all the mythological and semi-historical references (see Sections 47-48) occur in the "M" section. Indeed this section is peculiarly Hindu in contrast with the remainder of the manuscript.

III. The mathematical contents of the "M" section may be described as miscellaneous problems, which are generally solved by "rule of three", but a special feature is the occurrence of numerous "measures" and a special method of exhibiting their change-ratio. But, of course, these points in no way indicate work—rather otherwise for if section "M" were an entirely different work we might expect some duplication, and there is none here.

IV. The method of exposition is somewhat different. The example is followed by a statement and the answer is then given, generally, without any detailed working, and generally there is no "proof" or verification. There are, however, exceptions.

V. There are differences in language. Certain technical terms that are extremely common in the rest of the manuscript do not occur e.g., pratyaya and yuta.

An Indigenous Hindu Treatise

Dr. Hoernle believes that arithmetic and algebra developed in India entirely on indigenous lines. According to him "in the Bakhshali manuscript, there has been preserved to us a fragment of an early Buddhist or Jain work on arithmetic (perhaps a portion of larger work on astronomy), which may have been one of the sources.

(1) The symbols used here are merely mnemonic,
from which the later Indian astronomers took their arithmetical information." Again according to Hoernle, the arithmetic in India developed without any Greek influence, "Of the Jains, it is well-known, says Hoernle, "that they possess astronomical books of a very ancient type, showing no traces of western or Greek influence" (Here is a probable reference to the Suryaprajnapti.). In India arithmetic and algebra are usually treated as portions of works on astronomy. In any case, it is impossible that the Jains should not have possessed their own treatises on arithmetic, when they possessed such on astronomy. The early Buddhists too are known to have been proficient in mathematics. The prevalence of Buddhism in North-Western India, in the early centuries of our era, is a well-known fact."

Kaye, however definitely asserts that there is not the slightest evidence in the manuscript itself of its being connected either, with the Jains or Buddhists. It is Hindu (Saivite); the author was a Brahmana (fol. 50); to Siva, he attributes the gift of calculation to the human race (fol. 50); offerings to Siva are mentioned on more than one occasion (fols. 34. 44); references are made to certain incidents recorded and persons named in the Hindu epics (fol. 32 etc.) and there is not a single reference that could be construed as indicating any connection with Buddhism and Jainism.

Various Opinions about its Age.

The original Bakhshali work has been assigned various dates by several scholars. Hoernle says: "I am disposed to believe that the composition of the former (the Bakhshali work) must be referred to the earliest centuries of our era, and that it may date from the third or fourth century A. D. 1" This estimation about the age of the original Bakhshali work has been accepted as fair by eminent orientalists like Buhler and historians of mathematics like Cantor and Cajori. Thibaut has followed Hoernle in accepting the date of the present manuscript to lie lying between 700 and 900 A. D. But Kaye would refer the work to a period about the twelfth century. "The script, the language, the contents of the work", says he, "as far as they can give any chronological evidence,

(1) Indian Antiquary, XVII, p. 36.  (2) Indian Palaeography, p. 82.
(3) M. Cantor, Geschichte der Math, I. P. 598.
(5) G. Thibaut, Astronomie, astrologie und mathematik, p. 75.
Various Opinions about its Age

all point to about this period, and there is no evidence incompatible with it”\(^1\), Bibhutibhushan Datta agrees with that the work was written towards the beginning of the era, and he has discussed this issue in his paper, “The Manuscript” published in the Bulletin of the Calcutta Mathematical Society, 1929, XXI, p. 1. We shall present his arguments here.

Bakhshali Mathematics Older Than the Present Manuscript

Hoernle thinks that the mathematical treatise contained in the Bakhshali manuscript is considerably older than the present manuscript itself. “Quite distinct from the question of the age of the manuscript”, says he, “is that of the work contained in it. There is every reason to believe that the Bakhshali arithmetic is of a very earlier date than the manuscript in which it has come down to us.”\(^2\) This conclusion has been disputed and rejected by Kaye who thinks it to be based on unsatisfactory grounds. He then adds, “Of course it will be impossible to say definitely that the manuscript is in the original and only copy of the work but we shall be able to show that there is no good reason for estimating the age of the work as different from the age of the manuscript to any considerable degree.”\(^3\) Kaye has adversely criticised the linguistic and palaeographic evidence of Hoernle. Datta, however, thinks that Kaye’s arguments, if proved sound and sufficient, will establish that the present manuscript was written about the twelfth century, as is contended by him.\(^4\) Hoernle himself considers it to be not much older, belonging probably to a period about the ninth century of the Christian era.\(^5\) Most of the other reasons of Kaye against Hoernle’s view, based on certain internal evidence, such as (i) the general use of the decimal place value notation, (ii) the occurrence of the approximate square-root rule and (iii) the employment of the regula falsi, on imperfect knowledge of the scope and development

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\(^1\) Bakh. Ms., § 135.
\(^2\) Indian Antiquary, XVII, p. 36
\(^3\) Bakh. Ms. § 122
\(^4\) Bakh. Ms., § 135
\(^5\) Indian Antiquary, XVII, p. 36.
of Indian mathematics. There is, however, other internal evidence of unquestionable value to show that the Bakhshali mathematics cannot belong to so late a period in which Kaye would like to place it.

Bakhshali Work a Commentary

There is another noteworthy fact about the work contained in the present Bakhshali manuscript. From the method of its treatment, Hoernle thinks it to be a Karana work. In the opinion of Datta, the Bakhshali work is not a treatise on mathematics in its true sense, but a commentary—a running commentary, of course—on such an earlier work. The manner of its composition and particularly the very elaborate, rather over-elaborated details with which the various workings of the solution are most carefully recorded, without trying to avoid even unnecessary repetitions, strongly tend to such a conclusion. Here and there are given explanatory notes of passages, literary synonyms of words and technical terms, some of which will in no way be considered difficult, or which are already well established. For instance, on folio 3 verso the word parasparakrtam has been explained by the word gunitam (tata parasparakrtam gunitam); again on a subsequent occasion, this latter term has been interpreted as equivalent to another more difficult and less known term abhyasa (tatra guna abhyasam; folio 27, recto). On another occasion we have avrttipravrttigunanam, abhbhbh>bhbhbhbhbh (folio 12, recto). It is stated in several instances that

(1) Indian Antiquary, XII, p. 89.

(2) तत परस्परक्रमः गुणितं जातम् (folio 3, verso);
    गुणित जातम् | ६ | ्| प्रबृत्तिर्मितयः: (folio 15, verso)
    पृथक् ह्रयं विनिक्षिप्य । पृथक् ह्रयं जिन्त जातं...म्यासो तत्त गण
    ४ ४ । अब्ध्यासं । १२ । रूढ्दिनं । १ २ । आभागाचुपवंचका
    ४५ । अष्ट क्षित्तजातं
    (folio 27, recto)

(3) तदाभान्ततोगुरुः...ततः | २५ । आभावृत्तिप्रबृत्तिगणनं ततः | ६ | अनेन गुणितं
    जातं
    (folio 12, recto)
ksayampastum, शर्यकाः Here is a very typical passage from the work.

"...........Dvighnamadi' adidvigna | 2 | chayojjihum | cha (54)
uttaram ato uttaram | patayitva ekam bhavati ...."

The above style of composition is very characteristic of a commentary. And the whole work is written more or less in the same style.

Then are some cross-references in the Bakhshali Manuscript which are of immense significance. For instance it has been observed about the 10th sutra (rule), which refers to a method of multiplication, that “this rule is explained on the second page”. A similar remark has been made about the 14th sutra that it is “written on the seventh page”. The importance of these two observations in determining the character of the Bakhshali work cannot be overestimated. It will be easily recognized that those observations cannot in any way be due to the author of the original treatise. For evidently those sutras occur at two places in the work. No author is likely to retain consciously such recurrences in his work and pass them merely by giving a cross-reference. So the duplications, as also the observations, must be attributed to a second person, the commentator. And they happened in this way: The original Bakhshali treatise was not a systematic work. It was an ordinary compen-

(1) (i) कृत्वा रूप क्षय पास्तमिति
(ii) कृत्वा रूप क्षय पास्तमिति
(iii) कृत्वा रूप क्षय पास्त | २ | ३ | ४ | जातु संगुण्य जात
   | ३ | ४ | ५ |
(2) ब्रह्ममार्गाणि आदि ब्रह्मुणि | २ | च मोजितं | ३ च उत्तरं | अती उत्तरं पात्यित्वा एकं भवति******
(3) एवं सूत्रं विदीयपते विवर्तितः
(4) सप्तम******प्रेषितं लिखिता****रिष्य (हित)'
(5) केन कलेन सात्यतां | एवमेकावशं पतेश्चलितितं पूर्णिष्य
   —Folio 12, recto
   —Folio 10, verso
   —Folio 14, recto
   —Folio 50, verso
   —Folio 1, recto
   —Folio 3, recto
   —Folio 4, verso
dium of mathematical rules and examples in which the rules relating to the same topic of discussion even, were not always put together at the same place. We notice this irregularity of treatment to a certain extent in the portion of the treatise which has been left to us. It may be pointed out that such irregular treatment is not at all unusual in case of early works and we find another instance of the kind in the Aryabhatiya of Aryabhata (466 A.D.). The commentator very properly attempted to improve upon the order (rather disorder) of the author, here and there, as far as possible, without disturbing it too much, by noticing and commenting upon at the same time the sutras which are very closely connected. So that he had sometimes to explain a sutra earlier than its turn according to the plan of the author. Sometimes a commentator is compelled to refer to a subsequent sutra before time owing to indiscretion of the author. Therefore, when there comes the proper turn for the explanation of such a sutra, he simply passes it over, very naturally, by giving the cross-reference to previous pages. Thus there will remain very little doubt that the present Bakhshali work is a running commentary on an earlier work. Further there are found other cross-references which very strongly suggest that the illustrative examples are also due to the original author.¹

Present Manuscript—A Copy

Inspite of what is stated on the contrary by Kaye² there are many things to make one believe that the present manuscript is not the original of the Bakhshali work, but is a copy from another manuscript. For it exhibits writings of more than one scribe, possibly of five³. This can be explained most satisfactorily only on the

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¹ For instance, the author may give an illustrative example which may involve a mathematical principle which is yet to be explained. An instance of the kind is found in the Trisatika of Sridhara where the author very indiscretely gives two examples (Ex.7) in illustration of the Rule 16, which involves mathematical principle explained in the Rules 23 and 24. In this work the commentator, who is no other than the author of the treatise himself gives the cross-references.

² Bakh. Ms., p. 74 footnote. "There is evidence that the Ms is not a copy at all. It is not the work of a single scribe: there are cross-references to leaves of the Ms: there is a case of wrongly numbering Sutra and the mistake is noted in another hand-writing."

³ Bakh. Ms. p. 11, 97.
assumption that is a copy. Further on folio 4, verse, is found an
observation as regards a certain *sutra* (rule) that is a mistake
in the rule* (*sutra bhirantimastii).* The style of writing of this observ-
ation is same as that of other writings on the leaf. Utterly no doubt that all the writings on the leaf are due to the same
scribe. Moreover, though this observation is placed between two
lines of writings, it is not an ordinary case of interlining. From the
apportionment of space in and about the remark, it is apparent that
the remark was introduced at the time of making the copy, but not
on any subsequent occasion. Now that observation cannot be due
to the author of the original treatise. For no author would pass
over a mistake in his work with a mere observation that it is wrong.
So it must be from another person, possibly the scribe. There is
also another possibility, and there are reasons to believe it to be
more probable, that the scribe found it in the copy which he used.
In any case, it will follow that the present manuscript is a copy. A
more conclusive proof of this is furnished by the colophon that the
work is "written (likhitam) by a Brahmana mathematician, son of
Chajaka, for the education of the son Vasista." Had Chajaka been
the author of the work, the more appropriate and usual word for this
colophon to begin with would have been *krtam* or *viraecitam*
("composed").

The scribe seems to be a careless one. For the manuscript
is full of slips and mistakes. Here are a few of them:

1. On folio 4, verso, occurs the passage "*sodasamasutra 17*. 
Evidently the figure should be 16.

2. On folio 8, recto, a portion, "*uttarardhenabhajayet*" is

(1) *...विशिष्टपुजः*
सिक स्मायेव पुजा उपयोग्य भवतुः लिखितस्यज्ञकुडः गणकराज 
प्राप्तवेन। सर्वप्रवत्ते शास्त्राणां गणितं सूचिने तिथित। आशावादाने 
संसारे उत्तन पश्चास्त्रित्ते तदाकर्तुं शिवेन परमाश्मनुः पाठे १
सुपुनम् गणितं सत्ययागारणम्।
य च हीनः..." II—Folio 50, recto.

(2) गोडंशं सूर्यं १७—Folio 4, verso

* सूचीग्रांतिमिस्त
deleted¹. This was written by mistake for "uttarenabhajet" which is the relevant portion of the sutra meant for quotation there. The deleted portion can be traced to a preceding sutra (folio 7, verso), where we have "uttarardhena bhajitan".

3. On folio 11, verso, 158

\[
\begin{array}{c}
1 \\
5 \\
1 \\
64
\end{array}
\]

is twice miswritten for 158. This latter fraction is once again

\[
\begin{array}{c}
13 \\
64
\end{array}
\]

wrongly written as "158 to 1 se l". Another mistake on the leaf

64

is "93 to ...masa 9;" what is meant 93₁/₄.

(4) In the Bakhshali manuscript², the end of a sutra is usually marked by a special design. Owing to the carelessness of the scribe, the sign has been put many times at an intermediate place in the sutra. These illustrations are sufficient to show the carelessness of the scribe.

**Exposition and Method**

(i) The text consists of rules, sutras, and examples. There is no explanation whatever of the processes by which the rules were obtained, that is, there is no mathematical theory at all. In this the work follows the usual Indian fashion, as exhibited in all early texts. But there is a good deal of mathematical theory implied and the rules and examples are often set-forth in such a way as to convey the principles followed quite clearly to the student.

1. (उत्तराध्यन भाज्येत्), this is deleted in the MS; this was written by mistake for उत्तरेण भजेत्।

उत्तराध्यन भाज्येत्—Folio 7, verso

2. We have denoted this design by ⌨.
(ii) The rules (sutras) are written in verse and are generally numbered; and often in the solutions of the examples, the sutras are quoted. When Dr. Hoernle tried leaves of the manuscript, he took the numbered leaves of his order, but they were too few in number to lead to a satisfactory result. These sutras do not represent, as might be expected, the most valuable part of the text. They are usually of particular application rather than general and are often very obscurely expressed.

(iii) The examples given are generally formally stated without the use of notation or abbreviation of any kind; and in most cases they are stated in verse. They are introduced by the term *udu* an abbreviation for udaharanam" an example", After the question sometimes comes a formal statement with numerical symbols and abbreviations, often arranged in cells. Then comes the solution or working (karuna," and here, sometimes, fragments of the sutras are quoted. Finally comes demonstration—often more of the nature of verification than proof. Generally these demonstrations, by the aid of the answer found to the question, rediscover one of the original elements of the problem; and sometimes several such demonstrations are attached to an individual problem, but sometimes the variation is merely a matter of the form of statement.

The full scheme of exposition is therefore:

Sutram* or rule.

Udaharanam* or "example"; indicated its abbreviation udu* sometimes the example is called prasna*.

1. Folios 46, verso, and 65 recto.
2. Folios, 23, recto 25 verso; 29, recto, and 55, verso. Compare also folio 35, recto (a) and verso (b).
3. Folios 32, 36 44, verso and 46, recto. These references must have escaped the notice of Hoernle who remarks otherwise.

*9 (उद); *2 (उदाहरणम्); *3 (प्रशन); *4 (सूक्ष्म); *5 (उदाहरणम्); *6 (उद); *7 (प्रशन)।
Bakhshali Manuscript

Sthapanam or "statement". Sometimes it is also called nyasa or nyasa-sthapana.

Karanam or "solution".

Pratyayam or "verification". (Sometimes a solution is verified in more than one way.

The end of such sutra is marked after last example by the device and the number of the sutra is also given at the end.

The above will explain in general the method of the exposition of the Bakhshali work. But there are also occasional deviations from it. For it is not always that a sutra is illustrated by examples and an example is followed by its solution. There are at least two sutras in the surviving portion of the Bakhshali work which have no examples attached to them. They have been passed over as having been explained or written on preceding pages. Two examples are left without solution with similar remarks. Again solutions of examples in the beginning of the work have got no verifications, and so also a few others.

The above method of exposition differs considerably from what is now commonly met with in other Hindu mathematical treatises.

(1) For example, on folio 11, verso, there is mention of verification by the fourth method (anvam caturthapratyayam kryante)
(2) Vide folios 1—3: also folios 23—25.
(3) BrSpSi. ed. by Sudhakara Dvivedi Benaras, 1912, Solutions of some of the examples have been included as "authentic in Colebrooke"- Siddhanta (H. T. Colebrooke, Algebra with Arithmetic and Mensuration from the Sanskrit of Brahmagupta and Bhaskara, London, 1817) translation of the arithmetical and algebraical portion of Brahmagupta's. But they are not found in Dvivedi's edition of the work. Dvivedi also procured for himself a transcript of the copy of the commentary by Prthudakasvami which was originally secured by Colebrook (vide Dvivedi's Preface, p. 3). In these circumstances, we accept Dvivedi's version of Brahmagupta's above mentioned work to be more authentic. So it will have to be said that Brahmagupta did not give any solutions of his examples. See also the Brahasphuta-Siddhanta, New Delhi, 1968.

*२ (न्यास); *३ (न्यासस्वापन); *४ (कर्णम्); *५ (प्रत्ययम्)
Brahmagupta (628 A.D.) gives a very few examples (udaharana) in illustration of a limited number of his rules, but not their solutions. This want has been amply made up by his eminent commentator Prthudakasvami (860 A.D.) who has supplied sufficient number of illustrative examples with solutions under each rule. Muhavira (c. 850 A.D.) gives a copious number of examples for each rule. He calls them uddesaka*. But he too does not give solutions. The first writer to give statements (nyasa) as well as answers of his examples called udaharana is Sridhara (c. 750 A.D.). Then comes Bhaskara II. These writers have not recorded working of their solution. And no other Hindu writer is known to have given any verification of the solution of their examples. We do not find this even in the works of the latter commentators.

(iv) The method of grouping sets of figures is of interest, and shows features in common with mediaeval Sanskrit mathematical manuscripts, where also it is the practice to place groups of numbers in cells. In the Bakhshali manuscript, however, this fashion is rather more elaborated than in any Sanskrit manuscript so far examined. The mathematical possibilities of this scheme do not appear to have been realized and the student must always be careful to interpret any group of figures from the context, and not from any similarity with other groupings. However, there is a certain amount of consistency in the arrangements, as the examples exhibited below will show. The real purpose of the arrangements appears to prevent confusion by demarcating the numerical figures from the text itself. The text is

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(1) GSS., edited with English translation by T. Rangacharya Madras 1912.
(2) Trisatika, edited by Sundhara Dvivedi, Benaras, 1899.
(3) Lilavati edited by Sudhakara Dvivedi, Benaras, 1910, Bijaganita, edited by Sudhakara Dvivedi and revised by Muralidhara Jha, Benaras, 1927. English translation of these works are incorporated in Colebrook's Hindu Algebra.
often written almost independently of the figure groups, and a word may be arbitrarily divided by the cell arrangement, which may also cut into several lines of the text not necessarily connected with it. The economic necessity of utilizing the whole of the writing surface of the birch-bark available seems to have been the determining factor. The text itself should be consulted but the following examples may be helpful:

(a) Integral numbers occasionally occur without any marking off by lines or cells, but often.

(b) Each integral number has a cell to itself, e.g.

\[
\begin{array}{|c|c|c|}
\hline
1 & 42 & 39 \\
\hline
\end{array}
\]

(c) Sometimes an integer is marked off by two vertical bars: thus \(141\) and invariably a series of integers is thus demarcated, e.g.

\[
\begin{array}{|c|c|c|c|c|}
\hline
20 & 40 & 60 & 80 & 200 \\
\hline
\end{array}
\]

d) Fractions and groups of fractions are placed in cells or groups of cells, e.g.

(i) \(132\)

(ii) \(6055040625\)

(iii) \(11111\)

\[
\begin{array}{|c|c|}
\hline
33 & 3227520000 \\
\hline
\end{array}
\]

(iv) \(1\)

(v) \(40\)

\[
\begin{array}{|c|c|}
\hline
1 & 21 \\
\hline
3 & 21 \\
\hline
4 & 4 \\
\hline
5 & 5 \\
\hline
\end{array}
\]

(vi) \(1\)

(vii) \(20\)

\[
\begin{array}{|c|c|}
\hline
3 & 12 \\
\hline
4 & 12 \\
\hline
5 & 12 \\
\hline
\end{array}
\]

\(1\)
(e) Complete sets of operations are sometimes a similar manner. For example, (d) (iii) means (d) (iv) means \((1 \cdot \frac{1}{3}) (1 \cdot \frac{1}{3}) (1 \cdot \frac{1}{3})\); (d) (v) \(40 (1 - \frac{1}{3})\)

(f) Series of operations may be connected together by cell arrangements; for example:

<table>
<thead>
<tr>
<th>2 di^o</th>
<th>1 di^o</th>
<th>100000 di^o</th>
<th>phalam di^o 60000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>947</td>
<td>947</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3 di^o</th>
<th>1 di^o</th>
<th>157500 di^o</th>
<th>phalam di^o 60000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>947</td>
<td>947</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4 di^o</th>
<th>1 di^o</th>
<th>216000 di^o</th>
<th>phalam di^o 60000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>947</td>
<td>947</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

which means,

\[\begin{align*}
2 \frac{1}{2} \text{ dinaras} : & \quad 1 \frac{1}{2} \text{ days} : \quad \frac{100000}{947} \text{ dinaras} : \quad \frac{60000}{947} \text{ days} \\
3 \frac{1}{2} \text{ "} : & \quad 1 \frac{1}{3} \text{ days} : \quad \frac{157600}{947} \text{ "} : \quad \frac{60000}{947} \text{ "} \\
4 \frac{1}{2} \text{ "} : & \quad 1 \frac{1}{4} \text{ "} : \quad \frac{216000}{947} \text{ "} : \quad \frac{60000}{947} \text{ "}
\end{align*}\]

(g) The data of the problem and the solution may be indicated in one combined statement; e.g.

\[\begin{align*}
si^o 1 & \quad 16 & \quad 4 a^o \ che^o 2187* & \quad \text{phalam sara} 2624400 \\
1 & \quad 1 & \quad 1 & \quad 1 \\
1 & \quad 4 & \quad 1 & \quad 3 \\
\end{align*}\]

This is a statement of properties where the second term means
(h) \[
\begin{array}{c|cccc|cc}
& 0 & 2 & 1 & 3 & 3 & 12 & 4 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}
\] \text{ dr° } 300

may be roughly expressed by means of \( x + 2x + 3 \times 3x + 12 \times 4x = 300. \)

(v) The use in our text of the sexagesimal notation in the form in which it occurs is of rather special interest, for there is, as far as we know no other example of the kind in any of the classical Sanskrit works. The Hindus, from Aryabhata onwards, were well aware of the advantages of the sexagesimal notation for astronomical purposes, but they never used it for an arithmetical purpose.

Apparently there is only one purely arithmetical example of the use in the text and this example occurs, in connection with a problem in arithmetical progression, on folio 6, verso, and 7. recto, where the fraction \( \frac{178}{29} \) is expressed as \( 6 + 8' + 16'' + 33'' + 6iv. \) This sexagesimal fraction is actually written thus—

\[
\begin{array}{c|cccc|c}
6 & 8 & 60 & 16 & cha^\circ & \\
& 60 & 33 & li^\circ & 60 & \\
& 6 & vi^\circ & 60 & \\
& se^\circ & 6 & 29 & \\
\end{array}
\]

The upper three figures are missing in the manuscript but the restoration is certain. Of the abbreviation, \( li^\circ \) stands for \textit{lipta}, लिप्त (Gk. \textit{lipte}) which in Sanskrit words ordinarily means a minute of arc, or the sixtieth part of a degree; \( vi^\circ \) stands for \textit{villipta}, विलिप्त, ordinarily a second of arc; while \( se^\circ \) stands for \textit{sesam}, शेषम or "remainder".
Here one may not agree with Kaye that the purely (and perfectly legitimate) use of the notation points to influence (Kaye thinks that although such a use is Sanskrit works, it was extremely common in mediæval Muslim works).

It will be noticed that the term līpta here applies to "third parts instead of 'first parts', and vilipta to 'fourth parts'. The abbreviation cha° has not yet been traced to its origin.

(vi) The modern place-value arithmetical notation is employed throughout the text and Kaye says that there is not the slightest indication that to the author, it was a new or strange invention.

(vii) As already indicated algebraic symbols are not generally employed, but a symbol for the unknown quantity is. This symbol is the arithmetical symbol for 'nought' or 'zero' and on several occasions it is referred to by the term sunya-sthana (शून्यस्थान) or 'empty place'.

The symbol occurs some twenty times but only in sections II, C, F, G, H, K. Its employment is illustrated in the following example;—

(a) \[\begin{array}{cccccc}
\text{a°} & 1 & \text{u°} & 1 & \text{pa°} & 0 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}\]  
labdham 10

This is a 'statement' of an arithmetical progression where the first term is 1, the common difference is 1, the number of terms unknown, and the 'quotient' is 10 (where 10x=the sum of the series). Here the symbol simply indicates that the number of terms pada is unknown, i.e., that the place in the statement is empty. The symbol does not enter into any operation here or elsewhere. The giving it a denominator of unity is curious and really indicates that it is an integral number.

(b) \[\begin{array}{cccccc}
\text{a°} & 5 & \text{u°} & 6 & \text{pa°} & 0 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\text{a°} & 10 & \text{u°} & 3 & \text{pa°} & 0 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}\]  
dha° 0
Here are two equivalent arithmetical progressions in which the number of terms are equal and the sums also are the same in both cases, but both are unknown.

\[
\begin{array}{c|cccc}
\text{(c)} & 0 & 1 & 1 & 1 \text{ bha}^0 & \text{se} & 16 \\
1 & 1 & 1 & 1 & & 1 \\
3 & 3 & 3 & 3 & &
\end{array}
\]

which means \(x = 16/(1 - 1^3) \cdot (1 - 1^3) \cdot (1 - 1^3) \cdot (1 - 1^3)\) and since this is a deduction from a problem it is a distinct step towards a proper algebraic symbolism.

\[
\begin{array}{c|cccc}
\text{(d)} & 0 & 2 & 3 & 4 \text{ drsy}a & 200 \\
1 & 1 & 1 & 1 & & 1 
\end{array}
\]

which may be represented by \(x + 2x + 3x + 4x = 200\) But the method of solution is by the *regula falsi*. Any number is put in the place of the 0, and the sum of the series is obtained. Here the given sum multiplied by the assumed value and divided by the (false) sum is the correct solution...

\[
\begin{array}{c|cccc}
\text{(e)} & 0 & 5 \text{ yu mu} & 0 & \text{sà} & 0 & 7 \text{ mu} & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 7
\end{array}
\]

which means \(x + 5 = s^2, \ x - 7 = t\). Here the symbol \(0\) stands for three different unknown quantities. It simply indicates in each case an unknown number.

(viii) *Negative Sign*: The only distinct mathematical symbol employed is the sign for the negative quantity, which takes the form of a cross + placed after the number affected. This is peculiar and has given rise to discussions. In Sanskrit manuscript a dot before the quantity affected is the usual method of indicating a negative quantity. In the transliteration of the text the original negative sign (-) is, perhaps illogically preserved; but it is a rather special feature of the manuscript, easily printed, and leading to no ambiguity.

On this negative sign Dr. Hoernle writes (1)
“Here, therefore, there appears to be a mark of great antiquity. As to its origin I am unable to suggest any satisfactory explanation. I have been informed by Dr. Thibaut of Benares that Diophantos in his Greek arithmetic uses the letter $\phi$ (short for $\phi l_{\lambda}p_{\alpha}k_{\alpha}$) reversed (thus $\phi$) to indicate the negative quantity. There is undoubtedly a slight resemblance between the two signs; but considering that the Hindus did not get their elements or the arithmetical science from the Greeks (2), a native Indian origin of the negative sign seems more probable. It is not uncommon in Indian arithmetic to indicate a particular factum by the initial syllable of a word of that import subjoined to the terms which compose it. The only plausible suggestion I can make is, that it is the abbreviation ($ka$) of the word $kanita$, diminished”.

(ix) Abbreviations:—Abbreviations are employed to such an extent as to become, at times embarrassing, and they embrace almost every type of term. None of these abbreviations is used in an algebraic sense, although, at first sight, when we find $a^0$, $u^0$, $pa^0$ used consistently for the elements of an arithmetical progression, an algebraic symbolism does not seem very far off. But $a^0$, $ha^0$, $u^0$, $ga^0$ are the abbreviations of names of animals $ya^0$, $go^0$ and $sa^0$ of plants, etc., etc. The names of measures which are numerous, are nearly always abbreviated. Certain common terms of operation are often abbreviated, and of these the following occur most often—

bha$^1$ for bhaga$^2$, placed after a term to indicate that it is a divisor.

se$^03$ for sesam$^4$, a remainder.

mu$^05$ for mulam$^6$, a root, a quantity that has a root, capital.

pha$^07$ for phalam$^8$, an answer.

e tc.

Indeed the writers seem to have used abbreviations whenever there was no ambiguity incurred in their own minds. We are not so fortunately placed in this matter and occasionally it has been impossible to rediscover the term implied.

(2) They got a good deal from them, and particularly all their later astronomy came from the Greeks.
A. Illustration

Just to show how the problems are treated in the Bakhshali Manuscript, we shall reproduce one illustration:

(Folio I, verso and II, recto).

The *sutra* means change $\frac{a}{b}$ to $\frac{b}{b+a}$ and quotations from it are given on Folios 1, verso and 2, verso. The example solved on 1 verso and 2 recto is somewhat as follows:

The combined capitals of five merchants less one-half of that of the first, one-third that of the second, one-fourth that of the third one-fifth that of the fourth, or one-sixth that of the fifth is equal to the cost of a jewel. Find the cost of jewel and the capital of each merchant.

\[
\begin{array}{cccc}
120 & 90 & 80 & 75 \\
60 & 60 & 60 & 60 \\
\end{array}
\]

\[
\begin{array}{cccc}
72 & 90 & 80 & 75 \\
60 & 60 & 60 & 60 \\
\end{array}
\]

10. The sutra means change $\frac{a}{b}$ to $\frac{b}{b+a}$ and quotations from it are given on Folios 1, verso and 2, verso. The example solved on 1 verso and 2 recto is somewhat as follows:

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\begin{array}{cccc}
120 & 90 & 80 & 75 \\
60 & 60 & 60 & 60 \\
\end{array}
\]

\[
\begin{array}{cccc}
72 & 90 & 80 & 75 \\
60 & 60 & 60 & 60 \\
\end{array}
\]
Verso 1 appears to give part of the solution and proofs of the question on Folio 1, recto. Since.

\[ \sum x - \frac{1}{3} x_1 = \sum x - \frac{1}{3} x_2 = \sum x - \frac{1}{3} x_3 = \sum x - \frac{1}{3} x_4 = \sum x - \frac{1}{3} x_5 = C, \]

We have \( \frac{1}{3} x_1 = \frac{1}{3} x_2 = \frac{1}{3} x_3 = \frac{1}{3} x_4 = \frac{1}{3} x_5 = k \).

Whence

\[ \sum x = \frac{120 + 90 + 80 + 75 + 7}{60} = \frac{437}{60} k \]

\[ = \frac{437C}{60} \]

Folio 2, recto.
If \( k = 60 \), then \( C = 377 \), and \( x_1 = 120 \), \( x_2 = 90 \), \( x_3 = 80 \), \( x_4 = 75 \) and \( x_5 = 72 \).

Then follows a proof which may be expressed,

\[
90 + 80 + 75 + 72 = 317 \text{ and } 31 + 2 \cdot 39 = 377; \\
120 + 80 + 75 + 72 = 374 \text{ and } 347 + 3 \cdot 30 = 377; \\
120 + 90 + 75 + 72 = 375 \text{ and } 357 + 4 \cdot 30 = 377; \\
120 + 90 + 80 + 72 = 362 \text{ and } 362 + 5 \cdot 30 = 377; \\
120 + 90 + 80 + 75 = 365 \text{ and } 365 + 6 \cdot 30 = 377.
\]

Again on Folio 11, recto, there appears to be another verification of the example which we had on I, recto and verso.

\[
1 \frac{80}{3} + 90 + 80 + 75 + 72 = 377 \\
120 + \frac{80}{3} + 80 + 75 + 7 = 377 \\
120 + 90 + \frac{80}{3} + 75 + 72 = 377 \\
120 + 90 + 80 + \frac{75}{3} + 75 = 377 \\
120 + 90 + 80 + 75 + \frac{75}{3} = 377
\]

and this is the measure of the price of the jewel.

**Fundamental Operations**

In the classical Sanskrit works there is generally very little formal information about mathematical principles and methods. Axioms or postulates seldom or never occur and strict definition is seldom attempted. Rules are of particular application rather than general order appears to have been a matter of convenience rather than logic; and the fundamental operations receive scanty attention.

(i) Examples of *addition* (1)

(a) \( 960 \mid 64 \text{ yutam jatam } 1024 \)

(b) \( 10 \mid \text{ sa rupam } \mid 13 \)

\( 3 \mid 3 \)
Fundamental Operations

(c)  | 2 | rupa samyutam | 3 |

(d)  | 840 | 49 | datva jatam 889.

(e)  | 924 | 836 | 798 | esham yutim kriyate 2558

(f)  | 120 | 90 | 80 | 75 | 72 | esham yoga krite jata 437.

(g)  | 120 |  | 90 |  | 80 | 75 | 12 | evam 377

(h)  | 10 | 30 | 90 | ekatram | 130 | 1 |

(i)  | 29 |  | 48 |  | 58 | 7 anena yutam | 36 | 48 | 58 |

(k)  | 5 she 1 | 10 se  | 16 | evam 16 |

(l)  | 45 | sardha traya yutam | 52 | 2

(ii) Examples of subtraction (ii):

(a)  | 5 | 9 | vishesham | 4 |

(b)  | 3 | 2 | vishesham | 1 | 2 |

(1) (a) 960 and 64 added : 1024 is produced. (b) $\frac{1}{3}$ plus unity $= \frac{4}{3}$.
 (c) 2 and unity added together $= 3$. (d) 49 having been given to 840, 889 is produced. (e) 924, 836, 798 : the sum of these may be determined: they produce 2558. (f) 120, 90, 80, 75, 72 ; the sum of these is made : they produced 437. (g) 120, 90, 80, 75, 12 : thus 377. (h) 10, 30, 90 : altogether 130,
 (i) 29$\frac{4}{5}$ by this (7) increased $= 36\frac{4}{5}$.
 (k) 5$\frac{1}{2}$ $+$ 10$\frac{1}{6}$ thus 16. (l) $\frac{2}{3}$ with three and a half added $= 52/2$.

(ii) (a) 5, 9 : the difference is 4. (b) $\frac{3}{2}$, 2 : the difference is 1/2, (c) 5, 3 : subtracted, 2 is produced; (d) 6, 3 : the difference is 3; (e) 42 less three is 39; (f) 3, 7 : having subtracted, 4, (g) $\frac{7}{11}$, $\frac{9}{11}$ has having subtracted the difference is $\frac{9}{11}$.
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(c)  $| \begin{array}{c|c} 5 & 3 \end{array} | \begin{array}{c} \text{rahitam jatam} \end{array} | 2 |

(d)  $| \begin{array}{c|c} 6 & 3 \end{array} | \begin{array}{c} \text{shuddhi} \end{array} | 3 |

(e)  $| \begin{array}{c|c} 42 & \end{array} | \begin{array}{c} \text{tryunam} \end{array} | 39 |

(f)  $| \begin{array}{c|c} 3 & 7 \end{array} | \begin{array}{c} \text{vishodhya} \end{array} | 4 |

(g)  $| \begin{array}{c|c|c} 77 & 294 & 217 \end{array} | \begin{array}{c} \text{patya shesham} \end{array} | 12 |

(ii) Examples of multiplication:

(a)  $| \begin{array}{c|c} 2 & \end{array} | \begin{array}{c} \text{dvigunam}^2 \end{array} | 4 |$, i.e., $2 \times 2 = 4$.

(b)  $| \begin{array}{c|c} 30 & \end{array} | \begin{array}{c} \text{asta gunam} \end{array} | 240$, i.e. $30 \times 8 = 240$.

(c)  $| \begin{array}{c|c} 2 & 40 \end{array} | \begin{array}{c} \text{gunita jatam} \end{array} | 16$, i.e. $\frac{2}{5} \times 40 = 16$.

(d)  $| \begin{array}{c|c} 4 & 1 \end{array} | \begin{array}{c} \text{gunita jata} \end{array} | 27 |$, i.e. $(4 + \frac{1}{2}) (1 - \frac{1}{4}) = \frac{3}{4}$.

(e)  $| \begin{array}{c|c} 6 & 1 \end{array} | \begin{array}{c} \text{anena gunitam jatam} \end{array} | 4 |$, i.e. $6 \times (1 - \frac{1}{4}) = 4\frac{1}{4}$

(f)  $| \begin{array}{c|c} 2 & 3 \end{array} | \begin{array}{c} \text{gunita jatam} \end{array} | 2 |$, i.e. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

(g)  $| \begin{array}{c|c|c} 40 & \end{array} | \begin{array}{c} \text{phalam} \end{array} | 16$, i.e. $40(1 - \frac{1}{5}) (1 - \frac{1}{4}) (1 - \frac{1}{3}) = 16$

(h)  $| \begin{array}{c|c} 3 & 4 \end{array} | \begin{array}{c} \text{abhyasam} \end{array} | 12$, i.e., $3 \times 4 = 12$.

(i)  $| \begin{array}{c|c} 8 & \end{array} | \begin{array}{c} \text{atma-gunam} \end{array} | 64$, i.e. $8 \times 8 \times 64$. 
Fundamental Operations

(j) \[
\begin{array}{c|c|c|c}
880 & 964 & \text{gunita jatam} \\
84 & 168 & \text{} \\
\hline
880 \times 964 & 848320 & 14112 \\
\hline
\end{array}
\]
\[\text{i.e. } \frac{880}{84} \times \frac{964}{168} = 14112\]

(k) \[
\begin{array}{c|c|c|c}
737 & 178 & \text{anena gunita jatam} \\
58 & 29 & \text{} \\
\hline
737 \times 178 & 65593 & 841 \\
\hline
\end{array}
\]
\[\text{i.e. } \frac{737}{58} \times \frac{178}{29} = \frac{65593}{841}\]

(l) \[
\begin{array}{c|c|c|c}
108625 & 65600 & \text{pada-ghna} \\
\hline
108625 & 3227520000 & 59425 \text{ multiplied by number of terms} \\
65600. & & 49200 \\
\hline
\end{array}
\]
\[\frac{6455040625}{3227520000} = \frac{59425}{49200}\]

(m) \[
\begin{array}{c|c|c|c}
405280 & 44004 & \text{samgunya jatam} \\
38724 & 77448 & \text{} \\
\hline
405280 \times 44004 & 179945781120 & 2999096352 \\
38724 \times 77448 & & \frac{2999096352}{\text{}} \\
\hline
\end{array}
\]
\[\text{i.e., } \frac{405280}{38724} \times \frac{44004}{77448} = \frac{179945781120}{2999096352}\]

(iv) **Examples of Division**: Fraction quantities are expressed in the usual Indian way with the numerator written above the denominator with out any dividing line, and they are usually placed in cells. This indication of the operation of division often does away with the necessity for an explanatory word or phrase.

**Examples of division** :

(a) \[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
168 & 4 & 168 & 6 & 168 & 7 \\
\hline
\text{labdham 42} & 28 & 24 \\
\end{array}
\]
\[\text{i.e. } \frac{168}{4} = 42; \frac{168}{6} = 28; \frac{168}{7} = 24\]

(b) \[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
4 & \text{vibhaktam} & 1 & 35 & \text{gunitam} & 35 & 4 \\
\hline
\text{} & \frac{1}{4} & \text{} & \frac{35}{4} & \text{} \\
\end{array}
\]
\[\text{i.e. 4 divided } = \frac{1}{4}, \text{ which multiplied into } 35 = \frac{35}{4}\]

(c) \[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
10 & 3 & \text{vibhaktvam} & 10 & 3 & 1 \\
\hline
\text{} & \frac{10}{3} & \text{} & 1 & \text{} \\
\end{array}
\]
\[\text{i.e. 10, 3 having divided } = \frac{10}{3}\]

(d) \[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
447 & 29 & \text{dalita} & 447 & \text{} & \frac{447}{58} \\
\hline
\text{halved} = \frac{447}{29} & \text{} & \text{} & \text{} & \text{} & \text{} \\
\end{array}
\]
\[\text{i.e. } \frac{447}{29} \text{ halved } = \frac{447}{58}\]

(e) \[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
132 & 33 & \text{vartyam jatam} & 4 & 1 \\
\hline
\text{} & \frac{132}{33} & \text{} & \frac{4}{1} & \text{} \\
\end{array}
\]
\[\text{i.e. } \frac{132}{33} \text{ reduced gives 4,}\]
i.e. having described the denominator \( \frac{798}{1463} \) becomes 798

\[ 2558 - 1463 = 1095 \]
i.e., 2558 - 1463 = 1095, where 1463 is the denominator of the fraction.

i.e. by this (60) the known quantity (300) is divided and \( \frac{1}{60} \)
of 300 = 5

Square-root and Surds

The method of extracting square-roots of non-square numbers exhibited in the text is of special interest. The rule is much mutilated but as it is given on three separate occasions the fragments pieced together enable us to give the complete sutra 1

"In case of a non-square (number), subtract the nearest square number; divide the remainder by twice (the root of that number). Half the square of that (that is, the fraction just obtained) is divided by the sum of the root and the fraction 2 and subtract; (this will be the approximate value of the root) less the square of the last term." 2

(1) The rule is not preserved in its entirety at any one place in the mutilated manuscript of the Bakhshali mathematics that has come down to us. But from references and quotations one can restore the rule completely:

\[
\begin{array}{c|c|c|c}
21 & 20 & 21 \\
\end{array}
\]

(2) English translation of this rule given by Kaye (Bakh. Ms Sec. .68) is discarded as being wrong and meaningless. He has admittedly failed to grasp the true signification of the text. "The rule as it stands," observes Kaye, "is cryptic and hardly translatable." He further thinks that "no rule for 'second approximations' is preserved". It is not true, The method of  

\textit{Foot-note carried to next page}
Square-root and Surds

Then follows a note to the effect that "by means of thin oil an approximation to the proper root of a mixed quantity is found. . . . .".

The rule as it stands is cryptic and hardly translatable, but fortunately there are examples given in some detail, and these show that the rule was extended to "second approximations".

The rule means that the first approximation to

$$\sqrt{Q} = \sqrt{A^2 + b}$$

is $A + b/2A$ or $q_1$; but $q_1^2 - Q = (b/2A)^2 = e_1$.

No rule for 'second approximation' is preserved but there are several examples; and, of course, no fresh rule is really required, for

$$\sqrt{Q} = \sqrt{q_1^2 - e_1} - q_1 - e_1 / 2q_1 =$$

$$A + \frac{4A^2 + b^2}{8A^3 + 4Ab}$$

and the 'second error' may be indicated by

$$e_2 = \left[ \frac{e_1}{2q_1} \right]^2 = \left[ \frac{b}{2A} \right]^4 / \left( A + \frac{b}{2A} \right)^2$$

All the examples preserved belong to section 'C' and appear to be merely subsidiary to the solution of certain quadratic equations finding the third approximation is indicated in the second line of the verso. Indeed the only two difficult words in the sutra are samshista (समशिल्ल्द्ध) and krittikshaya (कृतिक्षय) We have interpreted the form as referring to "the square of the root and the fraction," That that is the interpretation aimed by the author will be corroborated by what is written in the folio 56, recto; while evaluating the surd $\sqrt{481}$.

21
20
400 (बल) 1 (विशिल्ल्द्ध:) 21 (भा)
21
441 2

Compare also folio, verso. Kriti-kshya (कृतिक्षय) literally means "less than the squares" or less the "square". It means that the square of the approximate value thus found less the square of the last term of it will be equal to the given quantity. The commentator has attempted an explanation of it on folio 57, recto but it is much mutilated. He says: kritikshayam kritam esamulam (कृतिकशय कृत एसमुल). Our explanation seems to be the only rational one possible.
arising out of, problems in arithmetical progressions. These problems have been fully worked out by Kaye in Sec. 68 and below are given merely a summary of the square-root evaluations. There are no less than six such applications of this rule in the Bakhshali manuscript.¹

(i) \(\sqrt{41} = \sqrt{36 + 5}\) and \(b_1 = 6\) \(\frac{1}{2}\) while \(e_1 = 1\) \(\frac{4}{5}\)

and \(q_2 = 6\) \(\frac{7}{8}\)

(ii) \(\sqrt{105} = \sqrt{100 + 5}\) and \(q_1 = 10\) \(\frac{1}{2}\); \(e_1 = 1\) \(\frac{\sqrt{2}}{2}\);

\[q_2 = 10\frac{1}{4} - \frac{1\sqrt{2}}{2 \times 10 \frac{1}{4} } = 10 \frac{81}{328} ; e_2 = \left(\frac{1\sqrt{2}}{2 \times 10 \frac{1}{4} }\right)^2\]

\[i = 107584\]

¹ Folios 57 and 64, verso; 45, recto; 56, recto and 56, verso: 45 and 46, recto. Note the expression mulam shlistakaranya (मूलम श्लिष्टकरण्या) or "the root by the method of approximation" folio 65, verso.

64 verso, 57, verso, and 57 recto: are all (except the last line) concerned with one example, the beginning of which is lost. The example is \([a = 1, d = 1, s = 5]\); therefore,

\[q_1 = \frac{6 \times 5}{12} = \frac{77}{12} \text{ and } t_1 = \frac{65}{24} \text{. Therefore.}\]

\[s_1 = \left[\left(\frac{65}{24} + 1\right)^{\frac{1}{2}} + 1\right] \frac{65}{24} = \left(\frac{1}{2} \cdot 24 + 1\right) \frac{65}{24} = \frac{89 \cdot 65}{48} = \frac{5785}{1152} = 5 \frac{25}{1152} = 5 + \frac{1}{8} \left(\frac{5}{12}\right)^2\]

The second approximation is introduced by the square-root rule (as previously on folio 55, recto) and is given by:

\[q_2 = 6 \frac{5}{12} - \frac{1}{2} \left(\frac{5}{12}\right)^2 \div 6 \frac{5}{12} = \frac{11858 - 25}{1848} = \frac{11833}{1848} \text{.}\]

and \(t_2 = \frac{1}{2} \left(\frac{11833}{1848} - 1\right) = \frac{1}{2} \cdot \frac{9985}{3696} = \frac{9985}{1848} \text{ and}\)

"this is the number of terms when the sum is five)" —Kaye, Bhakh, Ms. Part III, p. 181.*
Square-root and Surds

(iii) $\sqrt{481} = \sqrt{21^2 + 40}; q_1 = 21\frac{40}{8}; \quad e_1 = \frac{1}{8} \left( \frac{40}{42} \right)^2$

$= \frac{1600}{14112};$

$q_2 = 21 - \left( \frac{20}{21} \right)^2 \div 2 \times 21 \frac{20}{21} = \frac{219020}{9681}; \quad e_2 = \frac{1}{8} \left( \frac{40}{42} \right)^4 \div \left( 2 \times 21 \frac{40}{22} \right)^2$

$= \frac{160,000}{2,999,096,352}.$

(iv) $\sqrt{889} = \sqrt{29^2 + 48}; q_1 = 29\frac{48}{58}; \quad e_1 = \frac{1}{24} \left( \frac{48}{58} \right)^2$

$= \frac{24}{841}.$

(v) $\sqrt{336009} = \sqrt{579^2 + 768}; q_1 = 579\frac{384}{579};$

$q_2 = 579\frac{515,225,088}{777,307,500};$

$e_2 = \frac{1}{8d} = \frac{21,743,271,936}{7,250,483,394,675,000,000}.$

(vi) $\sqrt{339009} = 579 + \frac{384}{579} - \frac{\left( \frac{384}{579} \right)^2}{2 \left( 579 + \frac{384}{579} \right)}.$

There is not much doubt about the exegesis of the rule. It was neither connected with continued fractions, nor with the so called Pellian equation. Brahmagupta gave the converse of the rule, namely $(A+x)^2 = A^2 + 2Ax$ from which the square-root rule given in our text is immediately deducible. In this connection, Kaye has been of opinion that the square-root rule itself was not used by the Indians and was not even noticed by them until the sixteenth century. Indeed the Hindus had a very good practical rule of their own, which was given by Sridhara (Trishatika, 46) and Bhaskara II.
(Lilavati, 138), namely—"Multiply the quantity whose square-root cannot be found by any large square number, take the square-root of the product—leaving out of account the remainder—and divide by the square-root of the multiplier". For example, $\sqrt{41} \approx 6.403 + \frac{1}{1000} \approx 6.40313$.

The above approximate formula is now generally attributed to the Greek Heron (c. 200 A.D.)\(^1\), and it is restated by the Arab Al-Hassarar (c. 1175 A.D.) and other mediaeval algebraists\(^2\). But it was known, as has been shown elsewhere, to the second order of approximation, to the ancient Indians several centuries before\(^3\).

The values of these surds are summarised below:

<table>
<thead>
<tr>
<th>$\sqrt{Q}$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$q$ (approximately correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>6.41667</td>
<td>6.40313</td>
<td>6.403124</td>
</tr>
<tr>
<td>889</td>
<td>29.828</td>
<td>—</td>
<td>29.8161030</td>
</tr>
<tr>
<td>33609</td>
<td>579.664</td>
<td>57966615</td>
<td>579.65…</td>
</tr>
</tbody>
</table>

We have in the ‘Sulba sutras (Baudhayana, c 800 B.C.) the value of $\sqrt{2}$, and the other surds, not only obtained by geometric devices but also by the process of continued fraction. Kaye is awefully mistaken in asserting that the “Square-root” rule was not used by the Hindus and was not even noticed by them until the sixteenth century.

**Rule of Three**

The term trairasika*\(^1\) “relating to three quantities” occurs in the Bakhshali Manuscript about a dozen times—always employed in the sense of arithmetical proportion. Most often it occurs in the

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(1) T. Health, "History of Greek Mathematics", Vol. II, p. 324; Heron's time is uncertain. He may have lived in the 3rd century. A.D.


(3) Bibhutibhusan Datta "Hindu Contribution to Mathematics", Bulletin of the Mathematical Association, University of Allahabad, (1927-28), 269,
phrase *pratyaya trairasikena* or "proof by the rule of three" and the proportion is generally set out in the following manner:

\[
\begin{array}{c|c|c|c}
1 & 1 & 4 \\
3 & 1 & 1 \\
\hline
\end{array}
\]

which means \(1 : 3 : 1 \frac{1}{2} : 4 : 18\).

The term *trairasika* is orthodox but the term *phalam* is used throughout our text quite appropriately as equivalent to "answer", and is not applied to the second term of proportion as in the *Lilavati*.

There are very many examples of the 'rule of the three' used either as the direct method of solution or as a proof. There are also some very much damaged examples of what appears to be problems in so-called compound proportion. There is nothing of the nature of a theory of proportion discussed but several examples of the following principle occur—

If,

\[
\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n}, \quad \text{then} \quad \sum a = A, \quad \sum b = B.
\]

**Calculation of Errors and Process of Reconciliation**

The Bakhshali mathematics exhibits an accurate method of calculating errors and an interesting process of reconciliation, the like of which are not met elsewhere. They are necessitated by the application of the foregoing approximate square-root formula. There are certain examples whose solution leads to the determination of the number of terms of an arithmetical progression whose number of terms is unknown but first term \((a)\), common difference \((d)\), and the sum \((s)\) are known. If the number of terms be \(t\), then according to the Bakhshali work,

\[
s = \left[ (t-1)\frac{d}{2} + a \right] t, \quad \text{...(1)}
\]
whence \( t = \frac{-[2a-d]+\sqrt{[2a-d]^2+8ds}}{2d} \)

The negative sign of the radical has been overlooked in the Bakhshali work. Putting \( p = 2a-d \) and \( Q = [2a-d]^2+8d \), we have,

\[ t = \frac{-p+\sqrt{Q}}{2d} \]

or \[ 2dt + p = \sqrt{Q} \]

also \[ 2s = t^2d + pt \] \( \ldots(2) \)

Oftentimes in the examples given the value \( \sqrt{Q} \) does not come out in exact terms, so that a method of approximation has to be adopted. Let \( q_1, q_2, \ldots, \) be the successive approximations to the value of \( \sqrt{Q} \) and let the values of \( t \) obtained from them be \( t_1, t_2, \ldots \). Neither of these values will evidently give the original quantity \( s \) when substituted in the equation \([1]\) for the purpose of verification of the results obtained. Suppose the values of \( s \) corresponding to the values of \( t \) be \( s_1, s_2, \ldots \).

Then,

\[ 2s_1 = t_1 2d + pt_1 \]

or \[ 8ds_1 + p^2 = \left(2t_1d + p\right)^2 \]

and \[ 8ds + p^2 = (2td + p)^2 \]

Therefore,

\[ 8d(s_1-s) = (2t_1d + p)^2 - (2td + p)^2 \]

now, \[ 2td_1 + p = q_1 \]

---

(1) The Kriti-ksaya (क्रितिक्षय) probably refers to this second error.
Hence, \( s_1 - s = \frac{q_1^2 - Q}{8d} \).

Since \( \sqrt{Q} = \sqrt{a^2 + r} = a + \frac{r}{2a} = q_1 \)

to the first approximation,

we have, \( q_1^2 - Q = \left( \frac{r}{2a} \right)^2 = \epsilon_1 \), say

Then \( \epsilon_1 \) will denote the first error. Therefore,

\( s_1 - s = \frac{\epsilon_1}{8d} \)

Similarly, for the second approximation, the error will be \( \epsilon_2 = \left[ \frac{(r/2a)^2}{2(a + r/2a)} \right] \)

and \( s_2 - s = \frac{\epsilon_1}{8d} \)

We shall now refer to a specific instance, in which

\( a = 1, \quad d = 1, \quad s = 60. \)

---

(1) Folio 56, verso and 64, recto. Portions of the detail workings are not preserved in the existing manuscript. But they can be easily restored.
Folio 56, verso and recto as preserved are,

| 880 | 964 | गुणित जाति | 848320 | चत्वारिंश पृथक्क प्रान्त वग्न | 14112 |
| 84168 | | | | चत्वारिंश पृथक्क स्थाने वग्न |
| 1600 | | एष उपरांतायक वेबन् | 846720 | कल्यंवत्ति | 14112 |
| | | | | कल्यंवत्ति | 60 |
| | | | | कल्यंवत्ति | 60 |
| | | | | कल्यंवत्ति | 60 |
| | | | | कल्यंवत्ति | 60 |

अक्षरे विलकल कृत्यूपायः केषफश्रेष्ठो वि संगुणम्।
तद् वग्न दल अक्षरे। हुति पुरुष हुति व्रत: ||
केषफश्रेष्ठो विसंगुण हुः... (carried over to next page)
The detailed workings given are

\[ 8ds = 480, \quad 2a - d = 2.1 - 1 = 1.480 + 1 = 481 \]

\[ \sqrt{481} = 21 \frac{40}{42} = \frac{882 + 40}{42} = \frac{922}{42}. \]

Then, \( t_1 = \frac{1}{2} \left[ \frac{922}{42} - 1 \right] = \frac{880}{84}. \)

Hence,

\[ s_1 = \frac{t_1(t_1 + 1)}{2} = \frac{880}{84} \times \frac{964}{168} = \frac{848320}{14112} = 60. \]

And

\[ \frac{e_1}{8d} = \frac{1}{8} \left( \frac{40}{42} \right)^2 = \frac{1600}{14112}. \]

\[ s = s_1 - \frac{e_1}{8d} = \frac{848320}{14112} - \frac{1600}{14112} = \frac{846720}{14112} = 60. \]

Again for the second approximation,

\[ \sqrt{481} = 21 \frac{20}{21} - \frac{(20/21)^2}{2(21 + \frac{21}{21} \cdot \frac{21}{21})} = \frac{425042 - 400}{19362}. \]

\[ = \frac{424642}{19362}. \]
Calculation of Errors and Process of Reconciliation

\[ t_2 = \frac{1}{2} \left( 424,642 \cdot 405,280 \right) = 38,724. \]

Hence,

\[ s_2 = \frac{t_2(t_2 + 1)}{2} = \frac{405,280 \cdot 444,004}{38,724 \cdot 77,448} = \frac{179,945,941,120}{2,999,096,352} \]

\[ \frac{\varepsilon_2}{8d} = \frac{404}{3 \times 21^4 	imes (21 \frac{3}{2})^2} = \frac{160,007}{2,999,096,352} \]

\[ s = s_2 - \frac{\varepsilon_2}{8d} = \frac{179,945,941,120 - 160,000}{2,999,096,352} = 60 \]

Negative Sign

In the Bakhshali manuscript, a negative quantity is denoted by a cross (+) placed after the number affected. Thus, 11 7 means 11—7. This is very remarkable. For in the manuscripts of Prithudakasvami (600 A.D.) and later Indian writers a dot is usually placed above the quantity for the same purpose, so that according to them 11—7 is denoted by 11 7°. The origin of the use of a cross for the negative sign has been the subject of much conjecture. Thibaut has suggested its probable connexion with the Diophantine negative sign \( \psi \) (reversed \( \psi \), abbreviation for \( \lambda \varepsilon \sigma \psi \), meaning "wanting"). This has been accepted by Kaye. But in the opinion of Datta such a conjecture seems to be hardly reliable. For firstly, the Greek sign or minus is not \( \psi \) but an arrow-head (↑) and "it is now certain", observes Heath, "that the sign has nothing to do with \( \psi \)." And arrowhead and a cross are too much different to be connected together, or too distinct to be confused for each other. Secondly, the Greek symbol itself is of doubtful origin. And above all, we are not sure if it is as old as it will have to be for being the precursor of the Bakhshali cross. For there is no manuscript of the Arithmetica of

(1) *Indian Antiquary*, XVII, p. 34
Diophantus 'which is older than the Madrid copy of the thirteenth century A.D. and again in many cases in this work, the negative quantity is indicated by writing the full Greek word for “wanting” in its different case ending. So we cannot be sure if Diophantus did actually use that symbol for the minus sign. Under such circumstances it will not be proper to assume the possibility of Greek connection for the negative symbol of the Bakhshali manuscript, Hoernle thinks it—though he is not quite confident in this respect—to be the abbreviation ka of the word kanita or nu (नू or una, अन) of the word nyuna, (न्युन), both of which means “diminished” and both of which abbreviations, in the Brahmi characters, would be denoted by a cross. Datta thinks that this supposition has got a very notable point in its favour. In the Bakhshali manuscript all other arithmetical operations are generally indicated by the abbreviations (initial syllables of the words of that import, though often the words are written in full and occasionally nothing is indicated at all). So it will be very natural to search for the origin of its negative sign in that direction. In this way, Hoernle’s hypothesis appears to be a very probable one. But its principal drawback is that neither the word kanita (कनित) nor the word nyuna (न्युन) is found to have been used in the Bakhshali work in connection with the subtractive operation. The nearest approach to that sign is that of ksha, श्र, abbreviated from kshaya (क्षय, “decrease”) which has been used several times, indeed more than any other word indicative of subtraction. The sign for ksha, (श्र), whether in the Brahmi characters or in the Bakhshali characters, differs from the simple cross (+) only in having a little flourish at the lower end of the vertical line. The flourish might have been dropped subsequently for convenient simplification.

\[3\) *Indian Antiquary*, XVII, p. 34.
\[4\) It may be noted that abbreviations of all sorts of things, mathematical as well as non-mathematical, have been freely used in the Bakhshali work (vide 62 of Kaye)

\[5\) अ, \(a^0\) (for asva, अश्व, horse).

\[6\) ह, \(h^0\) (for hasti, हास्त, elephant)

\[7\) उ, \(u^0\) (for ustra, उद्वत्र, camel)

(carryed over to next page)
Least Common Multiple

The plan of reducing fractions to the lowest common denominator before adding or subtracting is known correctly to the author of the Bakhshali mathematics. We have a few instances of its application in the work. In one instance, it is required to find the sum of the fractions.

\[ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6} \]

They are first reduced to a common denominator (सबूतं किर्मित) so as to become,

\[
\begin{align*}
120 & \quad 90 & \quad 80 & \quad 75 & \quad 72 \\
60 & \quad 60 & \quad 60 & \quad 60 & \quad 60 \\
\end{align*}
\]

respectively. Finally, the sum is stated to be \( \frac{437}{60} \).

<table>
<thead>
<tr>
<th>गो, go° (for go, gô, cow)</th>
<th>दी, di (for dinar, दीनार, a coin)</th>
<th>लि, li (for lipta, लिप्त, a minute of an arc)</th>
<th>वि, vi (for višpta, विश्प्त, a second of an arc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ऋ, य, pa° (for pada, पद, a term)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and similarly of plants. In mathematical operations, भ (bhu) for भाग, a divisor; शेष (she) for शेष, a remainder, मू (mu) for मूल, a root, and फ (pha) for फलं, an answer.

(1) अथे विभाग पद्यं पत्तं पत्तं पद्यं पद्यं च ।
| मूलं प्रतिपादित: सबूतं। किर्मित: आदि। |
| 120 | 90 | 80 | 75 | 72 |
| 60 | 60 | 60 | 60 | 60 |
| एवं योग कुले जात् 437 अतः सूच्यं 377 एवं मण्डलां मूलयं। |
Folio 1, recto and verso.
In a different instance, it became necessary to add up,

\[ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6} \]

It is stated that the sum, after having reduced to a common denominator (हरसम्बे कृते युत्), will be \( \frac{163}{60} \). On reducing the fractions.

\[ \frac{1}{2} \times \frac{5}{6}, \frac{2}{3} \times \frac{6}{6}, \frac{3}{4} \times \frac{6}{6}, \frac{5}{6} \times \frac{6}{6} \]

to a common denominator, they are stated to be respectively,

\[
\begin{array}{c|c|c|c}
924 & 836 & 798 \\
1463 & 1463 & 1463 \\
\end{array}
\]

---

(1) करण क्षय। संगृण्य कनका एप स्थापयते।

\[
\begin{array}{c|c|c|c|c}
1 & 2 & 3 & 4 & 5 \\
2 & 3 & 4 & 5 \\
\end{array}
\]

tतद् युतिभाजयेत् ततः हरसास्ये कृतं युतं | 163 | संयुतः:....

Folio 17, recto,

(2) उदाहरण। अन्योपर विदित विभव: संयुक्त: हयम्।

\[
\begin{array}{c|c|c|c|c}
7 & 3 & 5 & | \\
12 & 12 & 6 & | \\
12 & 12 & 6 & | \\
\end{array}
\]

अन्यो समिश्रय समिश्रधैवेत् करण स्थितम्। एप् किर्यते

\[
\begin{array}{c|c|c|c|c|c}
19 & 7 & 11 & 12 & 4 & 6 \\
12 & 12 & 6 & 19 & 7 & 11 \\
\end{array}
\]

***जातमय**

\[
\begin{array}{c|c|c|c|c}
924 & 836 & 798 \\
1463 & 1463 & 1463 \\
\end{array}
\]

| 924 | 836 | 798 | एषां युति किर्येत्***जाता | 2558 | क्रृद्ध प्रोज्यं 1095 एतम्य समुत्यम्

Folio 2, verso.
A fairly difficult case is to simplify

$$\frac{13\frac{3}{5}}{3} + \frac{13\frac{1}{8}}{3} + \frac{1\frac{1}{3}}{4} + \frac{2\frac{1}{5}}{5} + \frac{12\frac{1}{3}}{8} + \frac{1}{\frac{1}{3}}$$

The result is correctly obtained as \(\frac{1807}{240}\)

---

(2) अन्यत्र

द्वारादास व्यक्ति भीजने द्वारा शासनमानव बुद्धि बिनेक दानवित्त कस्य तुः।

भाण्डृकर द्वारादास व जाराण पिंवत्तय वै।

एव व्यय समुपपती के \(X\) कार्य ब्रूहि पिंव्वत करणिविविवेन द्वारादासात्वर्णतता

\[
\begin{array}{cccccccc}
10 & 2 & 3 & 8 & 13 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 3 & 2 & 1 \\
2 & 3 & 8 & 4+2 & 3 & 5 & 2 & 3 & 1 \\
\end{array}
\]

भा 1 1 1 1 1 1 1 1 1 1 1 1
12 33 1 1 360 भाण्डा 1200 गुणितिति

Translation:

...and also twelve and a half in thirty three, and one third days for the best wine for the consumption of merchants. In the treasure house was stored twelve hundred. Say O Pandita, how long can this expenditure continue.

The statement means—

Daily income = \(\frac{10\frac{1}{3}}{2\frac{1}{3}} = \frac{30}{9}\)

Daily expenditure = \(\frac{13\frac{3}{5}}{3} + \frac{13\frac{1}{8}}{3} + \frac{1\frac{1}{3}}{4} + \frac{2\frac{1}{5}}{5} + \frac{12\frac{1}{3}}{8} + \frac{1}{\frac{1}{3}}\)

\[\begin{align*}
&= \frac{1807}{240} \\
\end{align*}\]

(The daily loss is, therefore, \(\frac{1807}{240} - \frac{9}{2} = \frac{727}{240}\))

and \(\frac{1200}{727+240} \times \frac{1}{360} = \frac{800}{727}\) is the period)
The method of finding the least common multiple is found in the *Ganita-Sara-Samgraha* of Mahavira (c. 850 A. D.) and probably also in Prithudakasvami’s commentary on the *Brahma-Sphuta-Siddhanta*, but not in works of Aryabhata, Brahmagupta and Bhaskara.

**Arithmetical Notation—Word Numerals**

The arithmetical notation generally employed throughout the Bakhshali work is the decimal place-value notation. This fact has been differently utilised by different writers. On the one hand, Hoernle and Buhler, who believe in the antiquity of the Bakhshali mathematics, consider it as evidence of the earlier date of the discovery of that notation by the Indians. On the contrary, Kaye, who believes in the non-Indian origin of the place-value notation and in its late introduction into India, considers its general adoption in the Bakhshali work as proof against the hypothesis of the previous writers about the early date of this work. It is now definitely known that Kaye’s notions about the origin of the place-value notation is wrong. It was invented in India about the beginning of the Christian era, probably a few centuries earlier. For this, one may consult the writings of Datta. But apart from that controversy it should be noted that the nearly exclusive application of the notation in the Bakhshali work is very much noteworthy inasmuch in almost all the available Indian mathematical treatises, save and except the *Aryabhata* of Aryabhata (499 A. D.), we find copious use of the word-numerals. There is, however, evidence to show that the author of the Bakhshali work did know the principle of the word numeral system of arithmetical notation. In it we find the use of the word

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(1) GSS. *Ganita Sara-Samgraha*, III, 56.


(3) *Indian Antiquary*, XVII, p. 38.

(4) *Indian Paleography*, p. 82.

(5) *Bakh. Ms.*, Sec. 131.

Arithmetical Notation—Word Numerals

The use of word numerals is as old as the Vedas. The first occurrence is in the Jyotisa Vedanga (c. 1200 B.C.) and second in the Chandah-sutra of Pingala (before 200 B.C.). Again, in speaking of a very large number,

2653296226447064994...83218

the Bakhshali mathematics writes:

Sadvimsasca tripancasa ekonatrimsa eva ca

Dvasa (st) sadvimsa catuhcatvalimsa saptatil

Catuhsastina (va)... msanamturam

Trirasiti ekavimsa asta....... pakam,

Clearly the principle of the word-numeral system has been followed in this instance. The only departure from its popular features lies in (1) the use of the number names in the place of the word-names and (2) the adoption of the left-to-right system in the arrangement of the figures. But, according to Datta, these features,

--- Folio 60, verso.

--- Folio 4, recto.

(4) Chandah—Sutra, VI, 34: VII, 2, 3.
(5) उदा० पद्विविषय निपद्यंवादेकों निविषय च।

--- Folio 58, recto.

(6) See Datta's papers in the Bangiya Sahitya Patrika, 1335 B. S. (1928), pp 8-30 on the comprehensive history of the origin and development of the word-numerals,
though not common, are not altogether foreign to the system. Once at least the author has followed the right-to-left sequence. For the compound word 'Catuh panca' (चतुर्पंच) has been used to denote once | 4 | 5 | and again | 5 | 4 | . (Folio 27, recto).

Regula falsi

The rule of "false position" or "supposition" is used in two ways in our text¹ in solving linear equations.

(i) \( f(x) = p \) is solved by assuming a value \( e \) for \( x \). This gives \( f(e) = p' \) and the correct value of \( x \) is \( ep / p' \).

An example is,

The amount received by the first is not known. The second receives twice as much as the first, the third thrice as much as the first two and the fourth four times as all the others. Altogether they receive 300. How much did the first receive?

Suppose the first receives one, then second receives 2, the third 9, and the fourth 48; or altogether they receive 60. Actually therefore, the first received \( \frac{300}{60} = 5 \).

(ii) If \( f(x) = p \), reduces to \( bx + c = p \) and \( f(e) = p' \) reduces to \( be + c = p' \), then \( x = (p = p')/(b + e) \), which appears to have been considered useful when it was desired to keep \( b \) and \( c \) unchanged.

For example, if \( x_1 + x_2 = a_1, x_2 + x_3 = a_2, x_3 + x_1 = a_3 \), we have \( 2x_1 + (a_3 - a_1) = a_3 \); and if in place of \( x \) we put \( e \) then \( 2e + (a_3 - a_1) = a_3 \) and the correct value of \( x_1 \) is \( (a_3 - a_3')/(2 + e) \).

This rule of 'false position' is interesting as being, in a way, a precursor of algebraic symbolism. It connotes the idea of an un-

---

¹ No rule is preserved in the text but Bhaskara II gives the following (Lilavati, 50):

"Any number assumed at pleasure is treated as specific in the particular question, being multiplied and divided, raised and diminished by fractions, then the given quantity, being multiplied by the assumed number and divided by the result yields the number sought."
known quantity and even of a symbol for that quantity (e.g., as in our text) but it does not embrace the notion of such a quantity being subject to operation or being isolated. As soon as we introduce algebraic symbols, the rule, as it were, disappears. Algebraically, the rule solves $bx = p$ by $x = ep/p'$, where $p' = be$, which shows that the transformation from $bx = p$ to $x = p/b$ was not conceived.

There is no algebraic symbolism in our text but it may be noted that Bhaskara gives both the regula falsi and also an early form of algebraic symbolism. Neither Aryabhata, Brahmagupta nor Sridhara gives this rule or makes use of the principle. On this, Kaye says that its occurrence in the Lilavati, therefore seems to indicate that it was introduced into northern India after the time of Sridhara (XIth cent.). Mahavira (IXth cent.), however, uses the method in rather a special way in connection with a geometrical problem.

But such a surmise according to Datta is wholly baseless. We shall sum up here the views advanced by Datta:

The rule of 'false position' has been applied in certain cases in the Ganita-Sara-Samgraha of Mahavira. For instance, for finding out unknown quantity (avyakta, arzn; ajnata अज्ञात), the sum of the various fractional parts of which is known, Mahavira says:

"The given sum, when divided by whatever happens to be the sum arrived at in accordance with the rule (mentioned) before by putting down one in the place of the unknown (element in the compound fractions), gives rise to the (required) unknown (elements) in (the summing up of) compound fractions".

We have a few other instances of this kind in the work. Further Mahavira has applied the method of supposition in solving certain geometric problems. Hence Kaye is not truly accurate when

---

(1) Ganita-Sara-Samgraha, VII. 112.
(2) Ganita-Sara-Samgraha III. 107. Compare the original expression in the work etc. with the passage त्रयों रूप रङ्घने तथ्य द्वन्द्व and similar other passages in the Bakhshali work [folios 25, verso and 26, recto; compare also folios 23, 23]
[3] Ibid III, 122, 122, 135--7
[4] Ibid. VII, 112, 221. This should be more accurately called the geometric prototype of the regula falsi of algebra.
he says: "Mahavira (9th century), however, uses the method in rather a special way in connection with a geometrical problem". In fact Mahavira has made more extensive use of the method in connection with certain algebraical as well as geometrical problems. Still it is quite true that he has not made as general use of the method as is found in the Bakhshali work, or even in the Lilavati. In any case, Kaye's hypothesis of the foreign import of the rule of false position into India after the eleventh century must be abandoned. It should be further noted that Mahavira (c. 850) is a contemporary of the earliest Arab algebraist to use that rule, namely, Al khwarizmi (c. 825). Hence it is quite certain that the Hindus have not taken the regula falsi from the Arab scholars, if they have done so at all from a foreign nation.

Known still earlier in India—

It should be observed that the rule of false position was reported to by the Arab and European algebraists at the early stage of development of their science when there were no symbols. It almost disappeared from amongst them, as it is bound to do so, with the introduction of a system of notations. It will be nothing unreasonable to expect that such had been the case with that rule in India too, if it was ever followed here. Now it is a well established fact that the Hindus reached Nesselmann's third and the last stage of development of the science of algebra long before all the other nations of the world. They invented a good system of notations

(1) Bakh. Ms., Sec. 72, This statement of Kaye followed by another of the same kind, "It (the regula falsi) occurs in no Indian work until the time of Mahavira" Sec. 134, will obviously contradict his previous statement, "Its occurrence in the Lilavati, therefore, seems to indicate that it was introduced into northern in India after the time of Sridhara [11th century]" Sec. 72. Thus it appear that Kaye is not sure of his own grounds.

(2) Certain problems in the Bakhshali work, the Lilavati [pp. 10 et. seq], the Trishatika [pp. 13 et seq] and GSS [IV, 5—32] which are of the same kind but differ only in detail, have been solved in the first two works expressly by the regula falsi, but not so in the other two works, thought in them the unknown quantity has been tacitly assumed to be one.

(3) It should be noted in this connection that while there are ample proofs in the writings of the early Arab scholars of their heavy indebtedness to Indian Mathematics it is still to be proved that the Indians took anything in return from them.
Known still earlier in India

by the beginning of the seventh century of the Christian era. It has been laid down by Brahmagupta (628 A.D.) that a thorough knowledge of algebraic symbols (varna) is an essential qualification for a good algebraist\(^2\). We find mention of a symbol for the unknown even in the *Aryabhatiya* of Aryabhata (499 A.D.). So the method of false position must have disappeared from India before that time, or it is at least bound to have been regulated to a very inferior position from that time. This will account for the absence of the method from the works of Aryabhata and Brahmagupta as well as for its limited application in the *Ganita-Sara-Samgraha* of Mahavira. There is now left no direct evidence from the Hindu source to show that that method was followed in India before the fifth century A.D. Unfortunately no Hindu treatise on arithmetic or on algebra which can be definitely referred to that period has survived and come down to this day with the exception of the Bakhshali manuscript. There is, however, external evidence. A mediaeval Arabic writer of note, possibly Rabbi Ben Ezra (b. 1095) refers the origin of the rule of false position to India\(^1\). And if our hypothesis about the origin of the term *yavat tavat* (यावत् तावत्) for the unknown in Hindu algebra be true, it is in all probability so, then there will remain very little to doubt that the rule was known in India much earlier.

The rupona method

There are several references to the rupona (रूपोने) method, and the phrases *rupona karanena* (रूपोने कर्णेन) or *pratyaya rupona karanena* (प्रत्यय रूपोने कर्णेन) occur. In all the cases in arithmetical progression according to the rule.

\[
s + \left( t - 1 \right) \frac{d}{2} + a \cdot t \]


\[\text{[2] Brahmagupta says: 'By the pulverizer, cipher, negative and affirmative quantities, elimination of the middle term, colours for symbols and factum, well understood, a man becomes a teacher among the learned and by the affected square' (Colebrook, Hindu Algebra, p. 325).}\]

\[\text{[1] Smith, D. B., } \text{History of Mathematics, II, p. 437, footnote 5}\]
The recurrence of the phrase *rupona karanena* (रूपोन करणेन) seems to imply that the rule in question began with the term *rupona* which corresponds to the \((t-1)\) of the formula. The rule is not preserved in our text but we find the following in the *Ganita-Sara-Sangraha* of Mahavira.2

"The number of terms is diminished by one, halved and multiplied by the increment. This when combined with the first term of the series and multiplied by the number of terms becomes the sum of all".

The rule is exemplified in two ways in our text of which the following are particular cases:

(i) \[
\begin{array}{|c|c|c|c|}
\hline
\text{rupona karanen} & 1 & 3^\circ & 1 \\
\hline
1 & & & \qquad 19 \\
\hline
\end{array}
\]

(ii) \[
\begin{array}{|c|c|c|c|}
\hline
\text{rupona karanen} & 1 & 3^\circ & 1 \\
\hline
1 & & & \qquad 19 \\
\hline
\end{array}
\]

Folio 9, recto, verso.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{rupona karanen} & 1 & 3^\circ & 1 \\
\hline
1 & & & \qquad 19 \\
\hline
\end{array}
\]

Folio 7, verso.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{rupona karanen} & 1 & 3^\circ & 1 \\
\hline
1 & & & \qquad 19 \\
\hline
\end{array}
\]

Folio 4, verso

\[
\begin{array}{|c|c|c|c|}
\hline
\text{rupona karanen} & 1 & 3^\circ & 1 \\
\hline
1 & & & \qquad 19 \\
\hline
\end{array}
\]

Folio, 5 recto.

(1) Is this a faint echo of Pythagorean style? Anatolius writes: The Pythagoreans state that their master, in connection with numbers that form a right-angled triangle, showed how such could be composed by means of unity'. i. e.,

\[
[\frac{1}{2} (a^2 - 1)]^2 + a^2 = \left[\frac{1}{2} (a^2 + 1)\right]^2
\]


(2) रूपोनोगच्छो द्वितीयः प्र चिन्तामिद तो मिश्रः।
प्रभवेन पदायस्तसमकलित्व भवति सर्वेषां।

GSS. II, 63.
which means: \( a = 1, \quad d = 1, \quad t = 19; \) by the rupona method, the answer is \( s = 10. \)

(ii) In section ‘C’ the application of the rupona method is always worked out step by step. For example, when \( a = 5, \quad d = 3, \quad t = \frac{178}{29}; \) then \( s = [(t-1)\frac{1}{2}+5] t; \) and the rupona method is applied as follows:

\[
(t-1) = \frac{149}{29}; \quad 3 \times \frac{149}{29} = \frac{447}{29}; \quad \frac{447}{29} \times \frac{1}{2} = \frac{447}{58};
\]

\[
\frac{447}{58} + 5 = \frac{737}{58}; \quad \frac{737}{58} \times \frac{178}{29} = \frac{65593}{841}
\]

\[-77\frac{836}{841}\]

The origin of that name is supposed by Hoernle\(^2\) and Knye\(^3\) to be lying in the fact that “the rule in question began with the term rupona \( (-a-1), \) i.e. one less] which corresponds to the \( (t-1) \) of the formula”. The term rupona literally means, “deducting one”. As the rule is not preserved in the available portion of the Bakhshali mathematics, it is not possible to verify this.
supposition. Kaye, however, points out that the rule has very nearly the same beginning in the Ganita-sara-samgraha\(^1\) of Mahavira: Ruponena gaccho duli krtah... (रूपोनेन गच्छो दली कृतः). According to Datta, the above interpretation of the origin of the term rupona karana, though not impossible, does not appear to be very convincing. The technical terms which are commonly used in the Bakhshali mathematics in connection with the arithmetical progression, such as adi, prabhava, caya, uttarapada dhana, etc., are all the same as in the other ancient Indian treatises; the name rupana karana is unique for it. It is not met with elsewhere. It is further noteworthy that no other term in the Bakhshali mathematics, or in any other Indian mathematical treatise, is known to have been formed in the same way, with the opening word rule.

It may be noted here that in the Bakhshali mathematics, the word rupa (रूप) occurs also with different significance than unity. For instance, we find\(^2\)

\[
\begin{array}{c|c|c}
\text{rupadhiram} & 3 & \text{esa kala...} \\
\text{rupona karanena phalam} & 21 & \text{dvitiyasya trainrasikena...}
\end{array}
\]

... | 1 | di\(\circ\) | 7 | 3 | di\(\circ\) | pha\(\circ\) ru\(\circ\) 21”

or ‘...divided becomes 2; ‘quotient plus 1 (rupa),’ this increased by 1 becomes 3; which times...by the ruponakarana, the result is ru\(\circ\) 21.” Of the second, by the rule of three, the result is ru\(\circ\) 21”.

---

(1) ii, 63.

(2) Folio 7, verso. The problem is

\[7t = \left(\frac{t-1}{2} + 3\right) t, \text{ whence } t = 2 \left(\frac{7-3}{4}\right) + 1 = 3.\]

By the rupona method, \(s = [(3-1) \frac{3}{2} + 3] t = 21\) and by the ‘rule of three’ 1:7::3:21.
In this passage, the number 21 has twice been marked abbreviated from rupa. Again in a sutra (folio 8, recto) related to an arithmetical progression, we find the passage *labdham rupa* viniudset, (लब्ध रूप विनिदसित), that is, ‘the quotient should be indicated as rupa’. Here again the term rupa seems to have a purely technical significance. There are other instances in which rupa does not mean one, but is used in connexion with an integer. Similar use of the word rupa is found in later Indian mathematical treatises where it denotes, besides 1, an integer or the integral part of a mixed fraction. I venture to amend the word *rupa* (रूप) to *rupam* (रूप). Then it will mean ‘making rupa’, which means ‘known or absolute number’, ‘known quantity as having specific form’. So rupana-karana (रूपन-करण) will mean ‘the method of making absolute number’, that is, ‘totalization’ or ‘summation’. This hypothesis will be strongly supported by the expression ‘rupana karana phulum rupam rupa 21’ (रूपन करण फलं रूपं रूप 21, or ‘by the method of making rupa the result is rupa 21’).

Proofs

To many solutions are attached proofs, which are generally introduced by the term pratyayam, प्रत्ययम्, ‘proof’ or ‘verification’. This is sometimes amplified into pratyaya-trairasikena (प्रत्यय तैरासिनेन).

--- Folio 21, recto.

**एकोनविशिष्टम | गःवो 10 | रूप 8 | ··· | ··· विविरितासिद्ध**

--- Folio 60, recto.

(1) Folios 21, recto; 60, recto; etc.

(2) See Br.Sp. Si, XII, 2; Trisatika, pp. 7 et seq; Lilavati, pp. 6, 7; Bija., pp. 2 et seq; (Colebrooke: Hindu Algebra, p. 149).

(3) Rupona (रूपन = रूपन) may be an archaic form of rupana (रूप).

(4) See Monier-Williams, *Sanskrit English Dictionary*, revised by Cappaller and Leumann, on rupa. Compare this use of the word rupa with its use in algebra in the sense of absolute known number in an equation.
'proof by the rule of three', or pratyaya-rupona-karanena "proof by the rupona method".

Sometimes the 'proof' seems to be merely a matter of rewriting a statement in another form, but generally the answer is utilized and one of the original terms of the problem is rediscovered. Occasionally the 'proof' consists of a solution of the original problem in another way, e.g., by steps; but sometimes it deals with a subsidiary aspect of the original problem. In certain problems approximations are employed and then the 'proof' may be described as a process of reconciliation. These 'reconciliations' entail a comparatively high degree of mathematical skill. Occasionally several different proofs are attached to a problem. The following are specimens.

(1) Problem $\frac{5}{3} t - 7 = \frac{5}{3} t + 7$

Solution $t = 2 \times 7/(\frac{5}{3} - \frac{5}{3}) = 30$.

Proof $3:5 :: 30:50, 5:6 :: 30:36$ and $50 - 7 = 36 + 7$

—Folio 61, recto.

(2) Problem—A gives $2\frac{1}{2}$ in $1\frac{1}{2}$ days, B gives $3\frac{1}{2}$ in $1\frac{1}{2}$ days, C gives $4\frac{1}{2}$ in $1\frac{1}{2}$ days.

In what time they have given 500 dinaras altogether?

---

(carrded over to next page)
Proof. 2:1:1:1
\[
\begin{align*}
&\frac{100,000}{947} : 60,000 : 947 \\
&\frac{157,000}{947} : 60,000 : 947 \\
&\frac{216,000}{947} : 60,000 : 947
\end{align*}
\]

Folio 22, recto.

(3) Problem. One earns \(e\) and spends \(f\) daily. How long will a capital of \(C\) last?

Solution, 
\[
t = \frac{500}{2\frac{1}{9} + 3\frac{1}{9} + 4\frac{1}{9}} = \frac{60,000}{947}
\]

And \(A\) gives \(\frac{100,000}{947}\), \(B\) gives \(\frac{157,000}{947}\), \(C\) gives \(\frac{216,000}{947}\) dinaras.

Folio 22, recto.

(4) Problem. If 7 are bought for 2 and 6 sold for 3, and the capital is 24, what will be the profit?

Solution. 
\[
p = \frac{C}{C - s - 1} = 24 (\frac{7}{3} + \frac{3}{9} - 1) = 18
\]

Proof. 2 : 7 : : 24 : 84 (the number of articles)

and 6 : 3 : : 84 : 42 (the total proceeds),

and 42 - 24 = 18.

or \(\ldots 1 : c : : C : n, \quad v : 1 : : n : C + p\)

and \(C + p - C = p\),

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(5) Problem. \(x(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5})=x-280\)

Solution. \(\frac{400}{2} = 200, 400 - 200 = 200\)
\(\frac{200}{4} = 50, 200 - 50 = 150\)
\(\frac{150}{5} = 30, 150 - 30 = 120\)
and \(400 - 120 = 280\).

—Folio 52, verso.

(6) Problem.

\((1 + \frac{1}{2})(2 + \frac{1}{2})(3 + \frac{3}{2})(4 + \frac{3}{2})(5 + \frac{1}{2}))\)

Solution = \(\frac{9}{8}\).

Proof. \(((\frac{9}{8} - 1)2 - 2)2 - 3 - 4)\frac{3}{8} - 5 - \frac{1}{2}\) = 0.

(7) Problem. Solve \(x + 5 = s^2, x - 7 = r^2\)

Solution. \(x = \left[\frac{1}{\frac{9}{8}} \left(\frac{5 + 7}{2} - 2\right)\right]^{\frac{3}{2}} + 7 = 11\)

Proof. \(11 + 5 = 40 ; 11 - 7 = 2\).

—Folio 59, recto.

(8) Problem. \(F = \frac{1}{2}(1) + \frac{1}{2}(2) + \frac{1}{2}(3) + \frac{1}{2}(4)\)

Solution. \(F = \frac{163}{10} = \frac{163}{600}\)

Proof. \(\frac{163}{60} :: 1 :: \frac{163}{600}\)

---

(1) अन्य द्वारा शतांश पाद्...द्वारा

शते स्थिति 150 अबापि पद्धतां न कृत्। एवं...

\[
\begin{array}{ccc}
1 & 1 & 1 \\
2 & 4 & 5 \\
1 & 1 & 1 \\
2 & 4 & 1 \\
2 & & \\
\end{array}
\]

पद्धती जातकरण कृत...280/ांशपूति 28 | भागः 40 | 28 | 28

धनु 280 मुख्यं जातं 400 एष फलं स्थिति

—Folio 52, verso.
or, since \( F = \frac{\sum f \times w}{\sum w} \), \( \Sigma w : \Sigma f \times w : w_1 : w_1 F \)

(9) Problem. \( Dt = \left[ \frac{(t-1) d}{2} + a \right] t \)

Solution. \( t = \frac{2(D-a)}{d} + 1 \)

Proof. By the rupona method, \( s = \left[ \frac{(t-1) d}{2} + a \right] t \)

and \( Dt = s \).

(10) Problem. \( s = \left[ \frac{(t-1) d}{2} + a \right] t \)

Solution. \( t = \sqrt{(2a-d)^2 + 8s - (2a-d)} + 2d \)

—Folio 65, verso.

Generally the solution is an approximation \( t_1 \) or \( t_2 \) depending on the method of evaluation of the surd quantity; and the rupona method gives \( s_2 \) or \( s_2 \), neither of which is the same as the original \( s \).

A process of reconciliation is therefore called for, and this is given by

\[ s - s' = \frac{e}{8d} \]

where \( s' \) is the approximation to \( s \) given by the proof and \( e \) may be termed the square-root error. Problem. \( a = 1, d = 1, s = 60 \). First solution, \( t_1 = \frac{\sqrt{(2.1-1)^2 + 480 - 1} + 2.1}{(21 \ 40 \ 42 \ 41 \ 40)} + 2 = \frac{880}{84} \)

—Folio 65, verso.

First proof. \( s_1 = t_1 \ (t_1 + 1) + 2 = \frac{880}{84} \times \frac{964}{168} = \frac{848,320}{14112} \)

\( e_1 = \left( \frac{40}{42} \right)^2 \) and \( s = s_1 - \frac{e_1}{8d} = \frac{848,320 - 1600}{14112} = 60 \)

—Folio 56, verso.
Second Solution. \[ t_2 = \left( \frac{424.642}{19362} - 1 \right) + 2 = \frac{405.280}{38724}. \]

Second Proof. \[ s_2 = \frac{405.280}{38724} \times \frac{444.004}{77448} = \frac{179,945,941,120}{2,999,096,352}. \]

Now \[ e_2 = \left( \frac{40}{42} \right)^2 + 4 \left( 21 + \frac{40}{42} \right)^2 \]

and \[ \frac{e_2}{8d} = \frac{160,000}{2,999,096,352}; \]

and \[ s = s_2 - \frac{e_2}{8d} = \frac{179,945,781,120}{2,999,096,352} = 60 \]

—Folio 64, recto.

(11) The problems in section ‘G’ are characterised by numerous proofs—as many as five separate proofs being attached to one example. The following is fairly typical:\

Problem. \[ x = 500(1 - \frac{1}{4})(1 - \frac{1}{5})(1 - \frac{1}{6})(1 - \frac{1}{7}) \]

Solution. \[ x = 158 \frac{5}{6}. \]

Proof (i). \[ x' (1 - \frac{1}{4})(1 - \frac{1}{5})(1 - \frac{1}{6})(1 - \frac{1}{7}) = 158 \frac{5}{6}, \]

whence \[ x' = 500. \]

This is written horizontally.

Proof (ii). \[ x' (1 - \frac{1}{4})(1 - \frac{1}{5})(1 - \frac{1}{6})(1 - \frac{1}{7}) = 158 \frac{5}{6}, \]

whence \[ x' = 500. \] This is written vertically.

(1) उदाहरण चुम्बिण्य मददग्य सुवर्ण शत-पञ्चकर्षम्।

... ... ... ... अद्व प्रक्षय =

| पलं दूल 500 || पुनरेव प्रस्तार कर्षम् |
|---|---|---|---|
| 0 | 158 | सु। | 500 |
| 1 | 1 |
| 1 | 5* तो। |
| 4* | 1 |
| 1 | 64 |
| 4* | 500 | 1 | 1 | 1 | 1 |
| 1 | 1+ | 1+ | 1+ | 1+ |
| 4+ | 4+ | 4+ | 4+ | 4+ |
| 100 | 100 |
| 100 | 100 |

शेष 158 तो। 1 शेष 1

—Folio 11, verso.
Proof (iii). $x' = \frac{158\frac{3}{4}}{(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})} = 500$.

Proof (iv)—By steps,

\begin{align*}
500 & + 4 = 125, \\
375 & + 4 = 93\frac{3}{4}, \\
281\frac{1}{4} & = 70\frac{5}{6}, \\
210\frac{1}{6} & = 52\frac{2}{4}, \\
158\frac{3}{4} & = 5284.
\end{align*}

The solutions are sometimes very detailed, proceeding most carefully step by step, so that they become expositions in general terms of the processes involved. Also they often give actual quotations from the rules—-to such an extent sometimes that the original wording of the rule can be reconstructed. Unfortunately, however, such helpful suggestions are more often missing than not: in many cases the particular portions of the manuscripts are lacking or damaged beyond repair. In some sections, e.g. section ‘M’, nothing but the bare answer is generally given, and in others the outline only of the working is given.

The following examples illustrate these remarks:

(i) The problem is $DT + Dt = (t-1) + a$ where $a$ and $d$ are respectively the first term and common difference of an arithmetical progression, and $t$ the number of terms, to be found. The rule is

$t = \frac{2(D-a) + d + \sqrt{2(D-a) + d^2 + 8DdT}}{2} + 2d$ and the particular example gives $D = 5$, $T = 6$, $a = 3$, $d = 4$. The actual working of the solution, translated as literally as possible, is as follows:

“The daily rate diminished by the first term;* the daily rate is five yojanas, 5; the first term is 3; their difference is 2: this doubled is 4: this increased by the common difference is 8; and squared is $\frac{8}{4}$ which is known as the ksepa quantity.* *Multiplied by eight* ; the fixed term (30) multiplied by eight is 240; and multiplied by the common difference—multiplied 960. *The quantity known as ksepa is added*. Now the quantity known as kshepa is 64, which added gives 1024. The root of this is 32; the quantity set aside is 8 and this added gives 40. *Divided by twice the common difference: *twice the common difference is 8—divided 5.”

* The phrases marked off by asterisks are quotations from the sutra or rule.

9
The phrases placed between asterisks are quotations from the rule. A perfectly literal rendering would not be very intelligible to the ordinary reader and one or two gaps have been filled in; but the translation is a perfectly fair representation of the original.

(ii) The problem may be represented by \( x(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{8})=x-24 \) and the solution is given as follows:

*Having calculated the loss on unity* the terms become \( \frac{3}{8}, \frac{3}{8}, \frac{3}{8} \), and these multiplied together give \( \frac{27}{64} \). This subtracted from unity gives \( \frac{37}{64} \), which, inverted and multiplied\(^1\) by the given amount 24, is \( \frac{9}{8} \) of 24 = 40.

(iii) Something travels 3 yavas a day. How long will it take to go 5 yojanas? Here the solution is indicated by the following proportion only:

\[
3 \text{ yao} : \frac{1}{360} \text{ years} : : 5 \times 4,608,000 : 21,333 \text{ years} 4 \text{ months.}
\]

Symbols for the Unknown

In the Bakhshali mathematics the unknown quantity is referred to by the symbol 0, which is called ‘sunya,’ शून्य (‘void’ or ‘empty’).\(^2\) Datta thinks that strictly speaking it is not a symbol for the

---

1. Literally “divided and multiplied” into. This is generally used to indicate division by a fraction.

2 (i) यह तीन स्थिर स्थिर वापसी कर दें। विशेष रूप से वापसी कर दें।

The problem of this type:

"A certain amount was given to the first, twice that to the second, thrice it to the third and four times to the fourth. State the amount given to the first and the shares of others, if the total amount given was 200."

The shares are represented by 0, 1, 2, 3, 4. (Here zero stands as a symbol for the unknown). Now having added 1 to the unknown (nought), the sum is 1+2+3+4 = 10, which shows the proper share of the first is 200/10 or 20, and the series is 20+40+60+80 = 200.

The proof by the rupona method gives,  

--- Folio 22, verso. ---
unknown as has been supposed by Hoernle\textsuperscript{1} and Kaye\textsuperscript{3}. For the same symbol has also been used for the ‘zero’ (sunya) for the decimal arithmetical notation. This is, indeed, its true significance. Its use in connexion with an algebraic equation, in a sense other than for arithmetical notation, is simply to indicate that the quantity which should be there is absent or not known\textsuperscript{8}. Hence its place in the equation is left vacant and this is clearly indicated by putting the sign of emptiness there. Or in short, the use of the truely arithmetical symbol for zero in an algebraic equation is a clear proof of the want of a symbol for the unknown in the Bakhshali mathematics. Correctness of this interpretation will be borne out by the facts\textsuperscript{[1]} that this symbol does nowhere enter into any operation, as it ought to have done had it been truely a symbol for the unknown, and\textsuperscript{[2]} that often times it is referred to as sunya, sthana or the ‘empty place’, proving thereby that nothing is in that place\textsuperscript{4}. This

\[
\left[ \frac{(4-1)^2}{2} + 20 \right] 4 = 200.
\]

2(ii)...च विद्युणः

\[
\begin{array}{c|c|c}
0 & 1 & 4 \\
1 & 2 & 6 \\
\end{array}
\]

यदक्ष्य विद्युः शून्ये...क्षा | 1 | तदवर्गः तु कार्येत्

\[
\begin{array}{c|c|c|c|c|c}
2 & 2 & 3 & 6 & 1 & 1...
1 & 1 & 1 & 1 & 1 & ... \\
\end{array}
\]

प्रशिक्षें 33 दृष्यं विचित्रेत् | 132 \# वर्यंजातं | 4 |

Folio 23, recto.

The example may be represented by

\[x + 2T_1 + 3T_2 + 4T_3 = 132,\]

where \(T_1, T_2,\) etc. represent the values of the first, second, etc. terms. Make \(x = 1,\) then the terms are \(1 + 2 + 6 + 24 = 33,\) whence \(x = \frac{132}{33} = 4\) and the series becomes, \(4 + 8 + 24 + 96 = 132\) (Kaye).

(1) Indian Antiquary, xii, p. 90; xvii, p 30
(3) Compare such expression as mulam na jnayate, सलं न जायले (folio 13, verso; 15 verso, prathamam na janami) प्रथमं ने जानामि (24, verso), padam na jnayate पवं न जायलें (54, verso), etc. in each case of which the ajnata, अजात (unknown) element has been indicated in the statement by sunya, शून्य.
(4) Folio 25, verso ; and folio 26, recto,
hypothesis will be further supported by the fact that the similar use of the ‘zero’ sign to denote the unknown element in the statement (nyasa) of problems is found in the arithmetics of Sridhara¹ and Bhaskara². Thus we have³,

\[ \text{आदि: } 20 \quad \text{उ} \quad \text{श्च्च: } 7 \quad \text{गणितम् } 245 \]

which is a statement of an arithmetical progression whose first term is 20, number of terms is 7, sum is 245 and whose common difference is not known. Both these writers have well-defined notations for the unknown, and do never use the cipher in this way in their treatises on algebra. But as the use of algebraic symbols is not permissible in arithmetic, they make use of the cipher to indicate that certain element in a problem is wanting. Of course, the cipher has wider use in the Fakhshali mathematics than in any of these works.

The lack of an efficient symbolism is bound to give rise to a certain amount of ambiguity in the representation of an algebraic equation, especially when it contains more than one unknown⁴. For instance, in

\[ \begin{array}{ccc|c}
0 & 5 & \mu & 0 \\
1 & 1 & & 1 \\
\end{array} \]

which denotes \( \sqrt{x+5} = s \), \( \sqrt{x-7} = t \), different unknowns will have to be assumed at different vacant places. Again in the statement⁵.

\[ \begin{array}{ccc|c|c|c|c|c|c}
\text{अा} & 5 & \text{उ} & 6 & \text{प} & 0 & \text{ध} & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array} \]

which refers to two arithmetical progressions whose first terms and common differences are different but known, and whose

---

¹ Trisatika, pp. 19 et seq.
² ‘Lilavati’, pp. 18 seq. This is not evident from Colebrooke’s translation of the works where the cipher has been replaced by the query.
³ Trisatika, p. 29.
⁴ Nearly similar difficulty and inconvenience were experienced by the Greek algebraists who had only one symbol for the unknown.
⁵ Folio 59, recto. Hoernle and Kaye are not right in thinking that this statement represents:
\[ x+5 = s^2 \text{ and } x-7 = t^3. \]
⁶ Folio 5, recto,
Sums and number of terms are equal but unknown, \( \frac{0}{1} \) stands in the place of two different unknowns\(^1\). To avoid such ambiguity, in one instance which contains as many as five unknowns, the abbreviations of ordinal numbers such as \( \text{pra} \), \( \text{tr} \) (abbreviated from \( \text{prathama} \), प्रथम ‘first’), \( \text{dv} \) (from \( \text{dvitiya} \), द्वितीय ‘second’), \( \text{ca} \) (from \( \text{caturth} \), चतुर्थ ‘fourth’) and \( \text{pan} \) (from \( \text{pancama} \), पञचम ‘fifth’) have been used to represent the unknowns; e.g.,

\[
\begin{array}{c|c|c|c|c|c}
9 & 7 & 10 & 8 & 11 & 9 \\
7 & 10 & 8 & 11 & 16 & 17 & 18 & 19 & 20
\end{array}
\]

which means,

\[
x_1 + x_2 = 16, \quad x_2 + x_3 = 17, \quad x_3 + x_4 = 18, \quad x_4 + x_5 = 19,
\]

\[
x_5 + x_1 = 20.
\]

The want of a proper symbol for the unknown eventually leads to the adoption of the method of ‘false position’ or ‘supposition’ for solution of algebraic equations. The solution generally begins with putting ‘any desired quantity’ (यद्यच्छ यद्रच्छ) in the vacant place\(^3\).

---

\(^1\) It is not easy to say what is intended to be implied by placing the unity below the cipher. It is supposed by some to be an indication that the unknown quantity will be an integer (Kaye, Bakh. Ms., § 60). Such a supposition is quite untrue. For in the instance cited while \( \text{dhana} \) is an integer (= 65), \( \text{pada} \) is a fraction (= 13/3).

\(^3\) Strange! This very statement has been quoted by Kaye just after the remark referred to. In certain instances, it is a mixed surd (vide folio 6 and 45, rectos).

\(^3\) Folio 27, verso. In one instance in Bhaskara’s \( \text{Bijaganita} \), initial syllables of the names of particular things have been used as symbols for the unknowns. (Colebrooke, \( \text{Hindu Algebra} \), p. 195; compare also p. xi. cf. अ थ ब्रामाणकाचार्याच्य बाचाकाचरा ब्रामाणकाच्याच; \( \text{Bija.} \), p 2,

\(^3\) \( \text{Yadrecha pinyase sunye} \) यद्रच्छ विन्यासे सून्ये or \( \text{yadreche vinyase sunye} \) यद्रच्छ विन्यासे सून्ये is ‘putting any desired quantity in the vacant place’ (Folios 22, verso, and 23, recto). On another occasion it is said \( \text{Kamikam sunye pinyastam} \), कामिकम सून्ये पिन्यासात or ‘the desired quantity is placed in the vacant place’ (Folio 21, recto and verso). We have also such expressions as \( \text{sunya-shane rupam dattava} \), सून्य श्चाने रूपं दत्तव or ‘putting one in the vacant place’ (Folios 25, verso and 26, recto; compare also folio 22, verso).
Plan of Writing Equations

In the Bakhshali mathematics two sides of an equation are written down one after the other in the same line without any sign of equality being interposed. Thus the equations,

\[ \sqrt{x+5} = s, \sqrt{x-7} = t, \]

appear as¹

\[
\begin{array}{c|c|c|c|c}
0 & 5 & 0 & 7 & 0 \\
1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

The equation,

\[ x+2x+3.3x+12 \cdot 4x = 300 \]

is stated as²

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
0 & 2 & 1 & 3 & 3 & 12 & 4 & \text{द्रश्य} & 300 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

Sometimes the unknown quantity is not indicated. Thus the equation

\[ \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 65, \]

is represented as³

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
1 & 2 & 1 & 1 & 1 & \text{द्रश्य} & 65 \\
3 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

This latter plan is followed in the arithmetical treatise of Sridhara and Bhaskara. According to the former⁴

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
1 & 2 & 1 & 1 & 1 & \text{द्रश्य} & 2 \\
6 & 12 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

means

\[ x - \left( \frac{x}{2} + \frac{x}{6} + \frac{x}{12} \right) = 2. \]

Bhaskara does not use the lines⁵. It will be noticed that in all the aforementioned works the absolute term is called drsya, (द्रश्य) meaning 'visible' which is sometimes abbreviated into dr (द्र). A distinction is sometimes made in its connotation in the different works. The problems in connection with which the above equations arise are of the same kind in all the works. But in the Bakhshali work, the term drsya (द्रश्य) refers to the 'gives' while in the other works it generally refers to the 'remains'⁶. There is, however, one

---

¹ Folio 59, recto.
² Folio 23, verso. See also folios 21, 23, 24, recto, etc.
³ Folio 70, recto and verso (C). See also folio 69, verso.
⁴ Trisatika, pp. 13 et seq
⁵ Lilavati, pp. 11 et seq.
⁶ The term drsya occurs also in the (GSS), (IV, 4) in the sense of "remainder".
instance in the Lilavati in which the connotation of drsya is exactly the same as in the Bakhshali work\(^1\). This term is closely related to rupa (रूप), meaning ‘appearance’, which is the same for the absolute term in the Indian algebra. We find thus the true significance of the Indian name for the absolute term in an algebraic equation. It represents the visible or known portion of the equation while its remaining part is practically unknown or invisible.

The above plan of writing equations differs much from the plan found in ancient Indian algebra in which, (1) two sides are usually written one below the other without any sign of equality and (2) the terms of similar denominations are written below one another, the terms of absent denominations from either sides being indicated by putting zero as its co-efficient (Datta).

Certain Complex Series

As already stated, the author of the Bakhshali mathematics is well acquainted with rule for the summation of series in arithmetical progression. Indeed he gives considerable importance to its treatment. There are instances of the geometrical progression in the work\(^2\). There are further elementary cases of a certain class of complex series, the law of formation of which is quite clear. If \(a_1, a_2, a_3, \ldots\) denote the successive terms of any series, we find series of the type\(^3\):

\[
Lilavati, \text{ p. } 11.
\]

\[
\begin{array}{cccc}
1 & 3 & 9 & 27 \\
81 & 329 & 1
\end{array}
\]

(1) \(t_1 + 3t_1 + 3^2t_1 + 3^3t_1 + \ldots = 329\)

Set \(t_1 = 2\), then the first series becomes 242 and the second 87, and the combined series, \(2 + \frac{15}{2} + \frac{48}{2} + \frac{147}{2} + \frac{444}{2} = 329\).

(2) \(t_1 + \frac{3}{2}t_1 + 3t_1 + \frac{15}{2} + \frac{48}{2} + \frac{147}{2} + \frac{444}{2} = 329\).

(3) The series of these types occur respectively on folio 22, verso ; 23, recto ; 23, recto and verso ; 25, verso and 26, recto ; 24, verso, and 25, recto ; 51, recto and verso.
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(1) \(a_1 + 2a_1 + 3a_1 + 4a_1 + \ldots + na_1,\)
(2) \(a_1 + 2a_1 + 3a_2 + 4a_3 + \ldots na_{n-1},\)
(3) \(a_1 + 2a_1 + 3(a_1 + a_2) + 4(a_1 + a_2 + a_3) + \ldots\)
(4) \(a_1 + (2a_1 \pm b) + \{3a_1 \pm (b+d)\} + \{4a_1 \pm (b+2d)\} + \ldots\)
(5) \(a_1 + (2a_1 + b) + \{3a_2 + (b+d)\} + \{4a_3 + (b+2d)\} + \ldots\)
(6) \(a_1 + (2a_1 + b) + \{3(a_1 + a_2) \pm (b+d)\} + \{4(a_1 + a_2 + a_3) \pm (b+2d)\} + \ldots\)
(7) \(a_1 + (a_1 + a_2 + a_3 + a_4 + a_5) \pm (b+d) + \{a_1 a_2 + (b+2d)\} + \ldots\)

Evidently the series (4), (5), (6) are obtained respectively from the series (1), (2), (3) with the help of the subsidiary series in arithmetical progression,

\[b + (b+d) + (b+2d) + \ldots\]

Systems of Linear Equations

Examples of the following type occur:

\[x_1 + x_2 = a_1, \quad x_2 + x_3 = a_2, \ldots \ldots \ldots x_n + x_n = a_n\]

where \(n\) is always odd. We have,

\[a_n = (a_2 - a_1) + \ldots + (a_{n-1} - a_{n-2}) + 2x_1;\]

and if we assume \(x_1 = p\),

then, \(a_n' = (a_2 - a_1) + \ldots + (a_{n-1} a_{n-2}) + 2p.\)

Subtracting we get, \(x_1 = p + (a_n - a'_n)/2\), which is the solution employed in the text.

The following examples occur

(i) \(x_1 + x_2 = 13, \quad x_2 + x_3 = 14, \quad x_3 + x_4 = 15.\)

The value of \(x_1\) is assumed to be 5, then by ‘subtraction order’ \(x_2 = 8, \quad x_3 = 6\) and \(x_3 + x_4 = 11.\) The correct values are therefore, \(x_1 = 5 + (15 - 11)/2 = 7, \quad x_2 = 6\) and \(x_3 = 8.\)

(ii) \(x_1 + x_2 = 16, \quad x_2 + x_3 = 17, \quad x_3 + x_4 = 18, \quad x_4 + x_5 = 19, \quad x_5 + x_6 = 20.\)

Here the value of \(x_1\) is assumed to be 7 and \(x'_5 + x'_6 = 16\) therefore the correct values are \(x_1 = 9, \quad x_2 = 7, \quad x_3 = 10, \quad x_4 = 8, \quad x_5 = 11.\)

The following are implied,

1. Since in these two particular examples the values of \(a_1, a_2,\) etc. are in arithmetical progression a, simpler solution would be \(x_1 = \frac{m}{2}\) where \(m\) is the mean of the series \(a_1, a_2\) etc.
(iii) \( x_1 + x_2 = 9, \ x_3 + x_4 = 5, \ x_3 + x_4 = 8. \)

(iv) \( x_1 + x_2 = 70, \ x_2 + x_3 = 52, \ x_3 + x_4 = 66. \)

(v) \( x_1 + x_2 = 1860, \ x_3 + x_3 = 1634, \ x_3 + x_4 = 1722. \)

(vi) \( x_2 + x_3 + x_4 + x_5 = 317 \)
    \( x_1 + x_3 + x_4 + x_5 = 347 \)
    \( x_1 + x_2 + x_3 + x_5 = 357 \)
    \( x_1 + x_2 + x_3 + x_4 = 365 \)
    \( x_1 + x_2 + x_3 + x_5 = 362 \)

and the following, which, however, is too mutilated to be sure of

(vii) \( x_1 + x_2 = 36, \ x_2 + x_3 = 42, \ x_3 + x_4 = 48, \ x_4 + x_5 = 54, \ x_5 + x_1 = 60. \)

The directions seem to indicate that we should cancel by six.

We then get,

\[ a'_1 = 6, \ a'_2 = 7, \ a'_3 = 8, \ a'_4 = 9, \ a'_5 = 10 \] of which the solution is \( x'_1 = 4, \ x'_2 = 2, \) etc., whence \( x_1 = 24, \ x_2 = 12, \ x_3 = 30, \) etc.

Further Examples

The next set of examples can be represented in the form

\[ \gamma x - x_r = c - d_1 x_1, \ \Sigma x - x_2 = c - d_2 x_2, \ldots \gamma x - x_n = c - d_n x_n. \]

If we set, \( a = 1 - d, \) these become,  

1. This can be arranged in the form \( y_1 + y_2 = 365, \ y_3 + y_3 = 347, \ y_3 + y_3 = 362. \)
   \( y_4 + y_5 = 317, \ y_5 + y_1 = 357. \) Solving this we get \( x_1 + x_2 = 210, \)
   \( x_2 + x_3 = 170, \ x_3 + x_4 = 155, \ x_4 + x_5 = 147, \ x_5 + x_1 = 192, \) which
   solved gives \( x_1 = 120, \ x_2 = 90, \ x_3 = 80, \ x_4 = 75, \ x_5 = 72. \) (See folio 1)

Also note that most of these six examples can be expressed in the form

\[ \Sigma x - x_1 = a_1, \ \Sigma x - x_2 = a_2 \ldots \ \Sigma x - x_n = a_n. \]

The examples given in the text are undoubtedly akin to the 'Ephanthera',
usually attributed to Thymaridas, which may be expressed by \( x_0 = \frac{\Sigma a - \Sigma x}{n-1} \)

where \( x_0 + x_1 = a_1, \ x_0 + x_2 = a_2 \ldots x_0 + x_n = a_n. \) In Aryabhata's 'Ganita'
is a similar rule \( x_0 = \frac{\Sigma d}{n-1} \)

where \( \Sigma x - x_1 = d_1, \ \Sigma x - x_2 = d_2 \ldots \)

\( \Sigma x - x_2 = d_2, \) which Cantor (Vorlesungen über der Geschichte der Mathematik i, 624) considers to be a modification of the rule given by Thymaridas.

See also P. Tannery Mémoires Scientifiques, Tome II, pp. 192-195.
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\[\begin{align*} 
\sum x - a_1 x_1 &= \sum x - a_2 x_2 = \ldots = \sum x - a_n x_n = c, \text{ and } a_1 x_1 &= a_2 x_2 = \ldots = a_n x_n = \sum x - c = k. 
\end{align*}\]

Therefore \[\sum x = \left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right) k = \frac{p}{q} k.\]

A solution is \(x_1 = \frac{q}{a_1}, x_2 = \frac{q}{a_2}, \ldots, x_n = \frac{q}{a_n}\) and \(c - p = q'.\)

There are four examples illustrating this process which may be tabulated thus:—

<table>
<thead>
<tr>
<th>S. No</th>
<th>(d_1)</th>
<th>(d_2)</th>
<th>(d_3)</th>
<th>(d_4)</th>
<th>(d_5)</th>
<th>(p-q)</th>
<th>(c)</th>
<th>(x_1)</th>
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<th>(x_3)</th>
<th>(x_4)</th>
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<td>914</td>
<td>836</td>
<td>798</td>
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<td>(iii)</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>1463</td>
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<td>161</td>
<td>41</td>
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<tr>
<td>(iv)</td>
<td>2</td>
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<td>17</td>
<td>6</td>
<td>3</td>
<td>1</td>
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</tbody>
</table>

Of these only fragments remain. For example, of the first we have:—

(1) Compare with this the treatment of the epanthem by Jamblichus

(Heath, Greek Mathematics, 1, 93-94)

(1) The nearest approach to this that I (Kaye) have came across in an Indian work is given by Mahavira in his GSS, VI. 239-240).

Five merchants saw a purse of money. They said one after another, by obtaining \(\frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\) of the contents of the purse I shall become three times as rich as all of you.

Let \(p\) be the value of the purse and \(x_1, x_2\) etc., the original capitals:

then \(3 (\sum x - x_1) = \frac{p}{6} + x_1, 3 (\sum x - x_2) = \frac{p}{7} + x_2\), etc. whence

\[11 \sum x = \left(\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}\right) p = \frac{6508p}{10080}, \text{ or} \]

\[\sum x = \frac{6508p}{110880}. \text{ If } p = 110880, \text{ then } x_1 = 261. \]

But al-Karkhi (XIIIth cent.) gave (iii, 6) the following:—

A certain sum is divided among three people, one-half being given to the first, one-third to the second and one-sixth to the third. But then one-half the share of the first, one-third the share of the second and one-sixth that of the third is pooled and shared equally by the three.

(Contd. on next page)
Further Examples

\[\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{120}{60} + \frac{90}{60} + \frac{80}{60} + \frac{75}{60} + \frac{72}{60} = 437\]

\[90 + 80 + 75 + 72 = 317, \quad 317 + \frac{120}{2} = 377\]

\[120 + 80 + 75 + 72 = 347, \quad 347 + \frac{90}{3} = 377\]

\[120 + 90 + 75 + 72 = 357, \quad 357 + \frac{80}{4} = 377\]

\[120 + 90 + 80 + 72 = 362, \quad 362 + \frac{75}{5} = 377\]

\[120 + 90 + 80 + 75 = 365, \quad 365 + \frac{72}{6} = 377\]

The second of these examples may be expressed in the form

\[x_1 + x_2 - (\frac{1}{3} + \frac{1}{4} + \frac{1}{5}) x_3 = x_2 + x_3 - (\frac{1}{3} + \frac{1}{5}) x_1 = x_3 + x_1 - (\frac{1}{3} + \frac{1}{4}) x_2\]

From this \[\frac{19}{12} x_1 = \frac{7}{4} - x_2 = \frac{11}{6} x_3\] and \[\frac{\sum x}{\Sigma x - c} = \frac{12}{19} + \frac{4}{7} + \frac{6}{11}\]

= \(\frac{2558}{1463}\). The solution given is \(\sum x = 2558\) and \(c = 2558 - 1463 = 1091\), whence \(x_1 = 924, x_2 = 836, x_3 = 798\).

Of the third, there is sufficient of the formal question preserved to enable it to be restored.

"One possesses seven horses, another nine mules (?), and a third ten camels. Each gives one of his animals to each of the others and then their possessions become of equal value".

In A.D. 1225 this problem became famous as it was one of those propounded to Leonardo of Pisa and solved by him.

If \(x_1, x_2, x_3\) be the shares and \(3c\) be the amount pooled, then,

\[x_1 = \frac{x_1}{2} + c = \frac{\sum x}{2}, \quad x_2 = \frac{x_2}{3} + c = \frac{\sum x}{3},\]

\[x_3 = \frac{x_3}{6} + c = \frac{\sum x}{6} \quad \text{and} \quad \sum x - x_1 = 2c, \quad \sum x - 2x_2 = 3c,\]

\[\sum x - 5x_3 = 6c, \quad \text{whence} \quad \sum x = \frac{47c}{7}. \quad \text{Make} \quad c = 7 \quad \text{and} \quad x_1 = 33, \quad x_2 = 13, \quad x_3 = 1.\]
Bakhshali Manuscript

The fourth example of similar form.

Mutilated Examples:

The following occurs on some very mutilated scraps that have been pieced together. It is now impossible to say how it originated or what its context was.

\[ x(x+y+z) = 60 \]
\[ y(x+y+z) = 75 \]
\[ z(x+y+z) = 90 \]

We have \((x+y+z)^2 = 225\) whence \(x+y+z=15\), and \(x=4, y=5\) and \(z=6\).

Solution in Positive Integers:

There is one nearly complete statement and solution of the following pair of equations:

\[ x+y+z = 20 \]
\[ 3x + \frac{3}{2}y + \frac{1}{2}z = 20 \]

of which the only solution in positive integers is \(x=2, y=5\), and \(z=13\).

This type of problem was a favourite in Europe and Asia in early mediaeval times. Later it was known as the 'Regula Virginiun', 'Regula Potatorum', etc. It was given by Chang-ch'iu-chien (sixth century A. D.)\(^2\), by Alcuin the Englishman (eighth century) and others. About 900 A. D. it was pretty fully treated by Abu Kamil al-Misri who gives some six problems, varying from three to five terms and attempts to find all the positive integral solutions\(^3\).

---

(1) This set of examples is introduced by a rule which simply means: change the fraction \(A/B\) into \(B/(B-A)\). This illustrates the fact that the sutras or rules often makes no pretence of indicating in any way the general theory: they are merely intended to be helpful.

(2) Yoshio Mikami.

Quadratic Indeterminate Equations

In the earlier Indian works problems of this type do not occur, but exactly the same example is given by both Mahāvīra\(^{1}\) and Bhāskara II\(^{2}\).

Quadratic Indeterminate Equations.

There are two types of quadratic indeterminates preserved.

First Type: (i) \(x + a = s^2\), \(x - b = t^2\).

The solution given may be represented by

\[x = \left[\frac{1}{2} \left(\frac{a+b}{c} - c\right)\right]^2 + b\]

which makes both \(x + a\) and \(x - b\) perfect squares. In the actual example preserved \(a = 5\) and \(b = 7\) and the solution is \(x = 11\), which is the only possible integral (positive) solution obtainable from the formula. No general rule is preserved but the solution itself indicates the rule. It proceeds by steps thus:

\[5 + 7 = 12, \ 12 + 2 = 6, \ 6 - 2 = 4, \ 4 + 2 = 6, \ 6 - 2 = 4, \ 4 + 7 = 11.\]

The value of this type of detailed exposition is here self-evident. It seems to have been almost necessitated by the absence of a suitable algebraic symbolism\(^{3}\).

Second Type: (ii) \(xy' - ax - by' - c = 0\)

---

(1) *GSS.*, vi, 152.
(2) *ViJa-Gan.*, 158.
(3) This type of equation occurs in many medieval works from the time of Diophantus onwards (c. g., see Diophantus, ii, 11ff; Brahmagupta, xviii, 84; Al-Karkhi, p. 63, etc.) and has here no very special interest beyond the indication it gives that the Bakhshali text followed the fashion. Dr. Hoernle, however, thought that it indicated a 'peculiar' connexion between the Bakhshali Ms. and Brahmagupta's work: and from this deduced that our text 'may have been one of the sources from whence the later astronomers took their arithmetical information.' (*Indian Antiquary*, xvii, 1888, p. 37).
The solutions which appear to be followed in the text are
\[x = (ab + c) - m + b, \ y = a + m; \text{ or } y = (ab + c) + m + a, \ x = b + m,\]
where \(m\) is any assumed number. The only example that is preserved\(^1\) is \(xy - 3x - 4y + 1 = 0\) of which the solutions given are 15 and 4, and 16 and 5, i.e.,
\[(3 \cdot 4 - 1) \div 1 + 4 = 15, \ 3 + 1 - 4; \ (3 \cdot 4 + 1) \div 1 + 3 = 16, \ 4 + 1 = 5.\]

Happy Collaboration of the East and the West

In chemistry, astronomy, mathematics, medicine and surgery, there was always in our history a happy collaboration in the advancements, in which all great nations of the East, Middle-East and the West took part. The Bakhshali manuscript in its original form (agreeing to Dr. Hoernle's suggestion) must have had its authorship in the early centuries of our era. From generation to generation, it must have been copied out with alterations and additions. By this time the western mathematics had incorporated almost all that could be traceable to Greek sources, and had assumed new forms and included some new notions also. On the whole the western medieval mathematics tended to become less rigorous and but more

---

1. Unfortunately the text 'folio 27, recto' is so mutilated that the correctness of the interpretation here given cannot be guaranteed. But the equation was well known to mediaeval mathematicians and has historical interest. Brahmagupta gave a general solution (xvii, 61) which, however, he appears to have thought unnecessary. Bhaskara also gives a general solution (Bija Ganita, 212-214) together with demonstration in both algebraical and geometrical forms; but he also on another occasion (ib. 208-209) gives an arbitrary solution. The general solution is also given by Al-Karkhi.

Mahavira gives (vi, 284) the equation in a different form, namely:

\[\left( a + \frac{1}{x} \right) \left( b + \frac{1}{y} \right) = ab + A \]
which reduces to \(xy - \frac{a}{A} x - \frac{b}{A} y - \frac{1}{A} = 0\) of which the solution is \(x = \frac{b}{A} + \frac{1}{m} \left( \frac{ab + A}{A^2} \right), y = \frac{a}{A} + \frac{m}{A} - \frac{1}{A}\); or \(y = \frac{a}{A} + \frac{1}{m} \left( \frac{a+b}{A^2} \right), x = \frac{b}{A} + m\). If we make \(m = \frac{ab}{A} + 1\) then \(x = \frac{b+1}{A}, y = \frac{a+ab+A}{A}\) which is the solution given by Mahavira.

His examples are: The product of 3 and 5 is 15, and the required product is 18 or 14. What are the quantities to be added or subtracted? i.e. (i) \((3+1/x)(5+1/y)=15+3\), (ii) \((3-1/x_1)(5-1/y_1)=15-1\). His answers are (i) 2 and 7, (ii) 6 and 17.
mixed than the mathematics of the classical Greece. Practical calculation (logistic) altogether supported the earlier pure arithmetic; mensuration took the place of pure geometry, and algebra slightly developed. Such changes are, indeed, indicated in the later Alexandrian works, and what we sometimes term degeneration had already set in there. The introduction of a place-value arithmetical notation made calculation easier and more popular, and more intricate arithmetical problems than those exhibited in the Greek Anthology, appeared in the later mediaeval text-books. The general body of popular mathematical knowledge became more diffused. (Identical problems occur in Chinese, Indian, Arabic and European text-books of a comparatively early period). Indeed the mediaeval mathematical works of Asia and Europe had so much in common that at first it seems almost impossible to pick out that which is definitely western or eastern in origin. In this connexion it should be remembered that the early astronomers (Aryabhata II, and Varaha-Mihira, an astronomer of Greek or middle-east origin, settled and naturalized in India,) were among the first to collaborate with Greeks in mathematical and astronomical learning; that later the Arabs, after sampling Indian works, entered into this collaboration; and that it was from the Arabs that Europe received once more the learning it had previously rejected. (Kaye)

We give here a summary chronological table of the period, that may serve to recall the chief mathematical writers and their works:

<table>
<thead>
<tr>
<th>Before Christ—Rgvedic period</th>
<th>Earlier than 4000 B. C.</th>
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<tbody>
<tr>
<td>Taittiriya Samhita</td>
<td>2000 B. C.</td>
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<tr>
<td>Satapatha Brahmana</td>
<td>1000 B.C.</td>
</tr>
<tr>
<td>Baudhayana Sulba Sutra</td>
<td>500 B. C.</td>
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<td>Vedanga Jyautisa, Lagadha</td>
<td>500 B. C.</td>
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<tr>
<th>2nd to 3rd Century Bakhshali Manuscript (original)</th>
<th>300 A. D.</th>
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<th>5th Century</th>
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<tr>
<td>Hypatia</td>
<td>d.415 A. D.</td>
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<tr>
<td>Proclus</td>
<td>410 — 485 A. D.</td>
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<tr>
<td>Boethius</td>
<td>b.470</td>
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<tr>
<td>Aryabhata</td>
<td>b.476</td>
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<tr>
<td>Century</td>
<td>Figures</td>
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<tr>
<td>6th Century</td>
<td>Bhashkara I 522</td>
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<td>Eutocius</td>
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<td>Damascius 529</td>
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<td>Dominus</td>
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<td>Chang Ch'iu-Chien 550</td>
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<td>Varahamihira 505—587</td>
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<td>Isidore of Seville 570—636</td>
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<td>7th Century</td>
<td>Brahmagupta b.598</td>
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<td></td>
<td>Fall of Alexandria 640/1</td>
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<td>Papyrus of Akhmin</td>
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<td>Bede b.735.</td>
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<td>8th Century</td>
<td>Muhammad b. Musa</td>
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<td>Alcuin b.804</td>
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<td></td>
<td>Sridhara 750</td>
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<tr>
<td>9th Century</td>
<td>Mahavira 850</td>
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<td>Tabit b. Qorra 835—901</td>
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<tr>
<td>10th Century</td>
<td>al-Battani</td>
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<td>Aryabhat II c. 950</td>
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<td>Avicenna</td>
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<td>Pope Sylvester II d. 1003</td>
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<td>11th Century</td>
<td>Albiruni visited 1017—1030</td>
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<td>India</td>
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<td>al-Karkhi</td>
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<td>Psellus</td>
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<td>Omar Khayyam b. 1046</td>
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<td>12th Century</td>
<td>Adeland at Cordova 1120</td>
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<td>Bhaskara II b. 1114</td>
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<td>Leonardo b. 1175</td>
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For the contents of this chapter the author expresses his indebtedness to the critical account of the Bakhshali Mathematics given by Kaye in his publications and to the critical paper of Dr. Bibhutibhushan Datta in the Bulletin of the Calcutta Mathematical Society, 1929.
APPENDIX I

WEIGHTS AND MEASURES

The measures exhibited in the manuscript are of rather special interest. As a whole they are Indian and the terminology is Sanskrit; but there are some Sanskritized western terms such as lipta, dramma dinara, satera employed. Most of the terms are well defined but the values of some are doubtful. Money measures, however, are, as in most early Indian works, very ill-defined and hardly show any differentiation from measures of weight.

Change ratios: The change ratios are often given with considerable care and elaboration, and are expressed in several different ways. The change ratio appears to be considered as a divisor for it is most frequently marked by the term chhedam which indicates the operation of division.

Examples are:

- chhe° 80 rakti°—su° i.e. 80 raktika = 1 suvarna
- chhe° 24 am°—ha° i.e. 24 angula = 1 hasta
- chhe° 2 gha°—mu° i.e. 2 ghatika = 1 muhurta
- chhe° 4608000 ya°—yo° i.e. 4608000 yava = 1 yojana

urdha chchhe° 768000 a°—yo°, i.e. 768000 angula = 1 yojana, and in this particular example the operation of multiplication is to be performed.

urdha chchhedam 108000 viliptanam raśi, i.e. 108000 vilipta = 1 raśi and multiplication is indicated.

adha chchhedam 2000 pa°—bha°, i.e. 2000 pala = 1 bhara and division is indicated.

Another form is illustrated in the following examples:

tolenasti’dhane 12, i.e. 1 tola = 12 dhana.
dhanenasti am° 4, i.e. 1 dhana = 4 andika
dinaranasti dhane 12, i.e. 1 dinara = 12 dhana
Abbreviations:

The Bakhshali abbreviation for weights and measures are usually as follows:

- Adh, अधः = Adhaka, आधक
- Am, अमः = Amsha, अम्श
- Am, अं = Andika, अंधिका
- Am, अं = Angula, अंगुल
- Bha, भा = Bhara, भार
- Dha, धा = Dhanaka, धानक
- Dha, ध = Dhanusha, धनुष
- Di, डि = Dina, दिन
- Di, द्री = Dinara, दीनार
- Dram, द्राम = Drama, द्रम्म
- Dra, द्रान्त = Drankshana, द्रान्तन
- Dro, द्रो = Drona, द्रोण
- Gav, गव = Gavyuti, गव्युति
- Gha, घ = Ghatika, घटिका
- Gum, गुं = Gunja, गुंजा
- Ha, हा = Hasta, हस्त
- Ka, का = Kakini, काकिनी
- Ka, का = Kala, कला
- Kha, खा = Khari, खारि
- Kro, क्रो = Krosa, क्रोश
- Ku, कु = Kudava, कुदव
- Li, लि = Lipta, लिप्ता
- Ma, मा = Masa, मास
- Ma, मा = Mashaka, माशक
- Sa, सा = Satera, सतेर
- Si, सि = Siddhartha, सिद्धार्थ
- Su, सु = Suvarna, सुवर्ण
- To, तो = Tola, तोला
- Va, वा = Varsha, वर्ष
- Vi, वि = Vilipta, विलिप्ता
- Ya, या = Yava, यव
- Yo, यो = Yojana, योजन

Time Measures

1 Varsha = 12 Masa = 360 Dina
1 Dina = 30 Muhurta = 60 Ghatika

Arc Measures

1 Rashi = 30 Ansha = 1800 Lipta = 108,000 Vilipta
1 Ansha = 60 Lipta (\(\lambda \in \pi \nu \eta\))
1 Lipta = 60 Vilipta
Money Measures

1 Suvarna = 1 \frac{1}{4} Dinara = 2 \frac{2}{3} Dramkshana = 16 Dhanaka = 80 Raktika

1 Dinara = 2 Dramkshana
1 Dramkshana = 6 Dhanaka
1 Dhanaka = 5 Raktika

Weight Measures

1 Bhara = 2000 Pala
1 Pala = 8 Tola
1 Tola = 2 Dramkshana
1 Dramkshana = 6 Dhanaka
1 Dhanaka = 4 Andika
1 Andika = 1 \frac{1}{4} Raktika
1 Raktika = 3 \frac{1}{3} Yava
1 Yava = 2 \frac{1}{3} Siddhartha
1 Siddhartha = 2 \frac{1}{2} Kala
1 Kala = 4 Pada
1 Pada = 4 Mudrika

Length Measures

1 Yojana = 2 Gavyuti
1 Gavyuti = 8 Krosha
1 Krosha = 1000 Dhanusha
1 Dhanusha = 4 Hasta
1 Hasta = 24 Angula
1 Angula = 6 Yava
Bakhshali Manuscript

Capacity Measures

1 Khari = 16 Drona
1 Drona = 4 Adhaka
1 Adhaka = 4 Prastha
1 Prastha = 4 Kudava
1 Kudava = 2 Prasriti
1 Prasriti = 2 Pala
Arithmetical Problems
INCORPORATED IN
The Bakhshali Text
A--7

It is possible that 52 recto gives parts of the solutions of the example on 51 recto which would make that page the reverse, (of course, doubtful). What is left of the solution means:

\[ x \left( \frac{1}{10} + \frac{1}{8} + \frac{1}{4} \right) = 57 \]

or

\[ \frac{19}{40} x = 57 \]

\[ \therefore x = 120. \]

A proof by the rule of three

\[
\begin{align*}
1 : 120 : : & 1 : 10 : 12 \\
1 : 120 : : & \frac{1}{8} : 15 \\
1 : 120 : : & \frac{1}{4} : 30 \\
\end{align*}
\]

ii. The example, which is continued on 52 verso, may be expressed by \( x \left( 1 - \frac{1}{2} \right) \left( 1 - \frac{1}{3} \right) \left( 1 - \frac{1}{4} \right) = x - 280, \) whence \( x = 400. \) A proof follows: \( 400 \times \frac{1}{2} = 200 \) and \( 400 - 200 = 200; \) \( 200 - 50 = 150; \) \( 150 - 30 = 120; \) and \( 400 - 120 = 280 \)

Again, \( \frac{1}{2} + \frac{1}{4} \left( 1 - \frac{1}{2} \right) + \left( 1 - \frac{1}{4} \right) \left( 1 - \frac{1}{2} \right) = \frac{1}{2} + \frac{1}{8} + \frac{3}{8} = 1. \)

A-9

29 a, b c recto. The problem and its solution here partly preserved may be represented as

\[ x_1 + x_2 = 13; \] \[ x_2 + x_3 = 14; \] \[ x_3 + x_1 = 15. \]

If \( x_1 = 5, \) then \( x'_2 = 8, \) \( x'_3 = 6, \) and hence \( x'_1 + x_1 - 5 + 6 = 11. \) From this, the correct value would be

\[ x_1 = 5 + (15 - 11) - 2 = 6 \]

whence \( x_2 = 6 \) and \( x_3 = 8. \)
The phrase "shodhayet kramat" recurs in the next example and is a quotation from a lost sutra.

29 d, b, c. verso. The example here given (continued on folio 27 verso) is formulated with exactly the same phraseology as the previous one. It may be represented by:

\[ x_1 + x_2 = 16; \ x_2 + x_3 = 17; \ x_3 + x_4 = 18; \ x_4 + x_5 = 19 \text{ and } x_5 + x_1 = 20. \]

If \( x_1 = 10 \), then \( x_2 = 6; \ x_3 = 11; \ x_4 = 7; \ x_5 = 12 \text{ and } x_5' + x_1' = 22 \)

Therefore, the correct value of \( x_1 = 10 - (22 - 20) + 2 = 9 \), and then \( x_2 = 7, \ x_3 = 10, \ x_4 = 8 \) and \( x_5 = 11 \).

The phrases "icchu' and "shodhayet kramat" are quotations from a lost sutra.

A—10

27 verso gives the answer of the problem given on folio 29 verso, namely \( x_1 = 9, \ x_2 = 7, \ x_3 = 10, \ x_4 = 8, \ x_5 = 11 \text{ and the sums of the pairs of } 16, 17, 18, 19, 20. \)

27 recto. Solution of a lost problem which may have been \( xy - 3x - 4y + 1 = 0 \), of which solutions are:

\[ x = \frac{3.4-1}{1} + 4 = 15; \ y = 3+1 = 4; \ x = 4+1 = 5; \ y = \frac{3.4+1}{1} + 3 = 16. \]

The quotations are from a sutra very much like the one that follows. The phrase "prathak rupam vinikshipya" "having added unity in each case", appears to be a quotation from a lost sutra.

A—11

The example is solved on 1 verso and 2 recto and appears to have been somewhat as follows:

The combined capitals of five merchants less one-half of that of the fifth is equal to the cost of a jewel. Find that cost of the jewel and the capital of each merchant.

A 11. 1 verso. This appears to give part of the solution and proofs of the question on 1 verso.
Then follows 'proof' which may be expressed by

\[ 90 + 80 + 75 + 72 = 317 \text{ and } 317 + \frac{1}{2} = 377 \]
\[ 120 + 80 + 75 + 72 = 347 \text{ and } 347 + \frac{2}{3} = 377. \]
\[ 120 + 90 + 75 + 72 = 357 \text{ and } 357 + \frac{3}{4} = 377. \]
\[ 120 + 90 + 80 + 72 = 362 \text{ and } 362 + \frac{7}{8} = 377. \]
\[ 120 + 90 + 80 + 75 = 365 \text{ and } 365 + \frac{9}{9} = 377. \]

Compare with sutra 11 on 1 recto.

\section*{A 12}

\subsection*{A 12.2 recto}
This appears to be another 'verification' of the example on 1 recto et verso; and means :

\[ \frac{1}{2} + 90 + 80 + 75 + 72 = 377. \]
\[ 120 + \frac{2}{3} + 80 + 75 + 72 = 377, \]
\[ 120 + 90 + 80 + \frac{7}{8} + 72 = 377. \]
\[ 120 + 90 + 80 + 75 + \frac{9}{9} = 377. \]

and 'this is the measure of the price of the jewel'.

\section*{A 13}

\subsection*{A 13}
Example : One possesses 7 horses (a\(\text{o} = \text{asva})\), another 9 horses (ha\(\text{o} = \text{hayu}) and a third 10 camels (u\(\text{o} = \text{ushira}). Each gives one of his animals to both the others and then their possessions are of equal value. It is required to find the capital of each merchant or the price of each animal. If thou art able, solve me this riddle.

\section*{B 1}

\subsection*{B 1.8 verso}

i. The sutra is partially reconstructed from the quotations in the solutions below.

ii. The example is : There are ten horses of which five are yoked at a time to the chariot. How many changes should there be in a journey of one hundred yojanas and how much will each horse do ?

The solution is \(5 \times \frac{1}{2} = 10\) stages and \(10 \times 5 = 50\). Mahavira gives a similar example (vi, 158).
Problems

"ravi-ratha-turagas sapta hi chatvaro'sva vahant dhuryuktaḥ yojana-saptati-gatyah ko vyudhab ko chaturyogah". It is well known that the horses of the Sun's chariot are seven. Four horses are yoked at a time. They have to perform a journey of 70 yojanas. How many times are they unyoked and how many times yoked?

B 2

"For a certain feast one Brahman is invited on the first day, and on every succeeding day one more Brahman is invited. For another feast, 10 Brahmans are invited on every day. In how many days will their numbers be equal; and how many Brahmans were invited? The use of the term labdham is here rather curious. The phrases labdham dvigunitam kritva, tathadvunam, uttaram vibhajitum and rupadhikam are probably quotations from a sutra.

B 2. 9 verso. The example probably meant: A and B start for a place 70 yojanas distant. A travelled at the rate of 1 yojana a day and B at the rate of 6. At what point on his return journey did B meet A?

Since, \( \frac{x}{1} = \frac{2\cdot70-x}{6} \), where \( x \) is the distance traversed by A, we have \( x = \frac{2\cdot70-x}{6+1} = 20 \) as given in the text, and since A travels at the rate of one yojana a day, this is also the time.

Proof by the 'rule of three'.

1 day : 6 yo° :: 20 days : 120 yo°, and 70 — 20 = 50 and 70 + 50 = 120. Also 1 day : 1 yo° :: 20 days : 20 yo°.

B 4

B 4. i. One goes at the rate of 5 yojanas for 7 days and then a second starts at the rate of 9 yojanas a day. When will they have traversed equal distances?

The phrase gatisyaiva vishesam ca is a quotation from sutra 15 (folio 3 recto) and purva gata is a reference to the same rule.

The solution is \( t = \frac{7.5}{9-5} = \frac{35}{4} \) days. 'Proof by the rule of three'

1 : : 5 : : \( \frac{55}{5} \) : \( \frac{175}{4} \) and 1 : 9 :: \( \frac{35}{4} \) : \( \frac{315}{4} \) = 175 = 35
θena travels at the rate of 18 yojanas in one day for a period of 8 days. A second goes at the rate of 25 yojanas in one day. Determine in what time.......

D 5

D 4 With some uncertainty:

A earns $\frac{3\frac{1}{2}}{2}$ drammas in 2 days, B earns $2\frac{1}{2}$ in 3 days. A gives B 7 drammas and this makes their possessions equal. How long had they been earning?

Since, $\frac{3\frac{1}{2}}{2} \cdot t - 7 = \frac{2\frac{1}{2}}{3} \cdot (t+7)$, we have $t = \frac{14}{\frac{7}{2} - \frac{8}{3}} = \frac{168}{11} = 15 \frac{3}{11}$ days.

And 3 days : 2$\frac{1}{2}$ drammas : $\frac{168}{11}$ days : $\frac{294}{11}$ drammas and

$\frac{294}{11} - \frac{77}{11} = \frac{140}{11} + \frac{77}{11}$ days : $\frac{140}{11}$ drammas

ii. Another example of the same kind.

D 6

D 6. 67 recto. i. The example seems to relate to a game at which a certain quantity was staked and eventually all lost. The statement means,

$$1+\frac{1}{2} (2+\frac{1}{2} (3+\frac{3}{2} (4+\frac{3}{2} (5+\frac{3}{2} \cdot \frac{3}{2}))) = \frac{9}{2}.$$  

E 1

E 1. The problem may have been something like this: The rates of purchase are one, two, three, four and six articles for one drama.

The cost of one of each would be $1+\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{9}{2}$, therefore the cost of 12 of each is 27 drammas, and the numbers of articles are 12, 6, 4, 3 and 2.
E 2

53 recto. The following conjectural restoration of the problem is offered:

One goes 1 1/2 yojana in a day and another 6 in 3 days. If the first had a start of 9 yojanas, when would the second overtake him?

53 verso. The following is merely a guess at the problem: One goes 18 yojanas in 96 days and another 27 yojanas in 108 days. The first starts from A and the second from B and the distance AB is 9 yojanas. When will they meet if they go only for 1/4 (or 35 ghatikas) of each day? (60 ghatikas = 24 hours).

E 3

58 verso. There is basis for the following restoration—A man earns 3 in one day, a young woman 1 1/2 in one day, and... 1 1/2 in one day. If 20 earn 20 mandas in one, let x, y, z be the numbers of each class. Then, \[3x + \frac{3}{2}y + \frac{z}{2} = 20\] mandas of which the only solution in positive integers is that given in the text, namely \(x = 2, y = 5\) and \(z = 13\). This problem is known as the Hundred Hens problem in China, and as the Regula Virginum, etc., in Europe.

F 1

F 1.22 verso I. This appears to be the beginning of a new section. This sutra is lost.

The problem was something like this: A certain amount given to the first, twice that to the second, thrice it to the third, and four times to the fourth. State the amount given to the first and the shares of the others, if the total amount given was 200.

The shares are represented by 0, 1, 2, 3, 4. 'Having added one to the nought' the sum is 1 + 2 + 3 + 4 = 10. Then the proper share of the first is \(\frac{200}{10} = 20\). Having added in this value the series becomes 20 + 40 + 60 + 80 = 200. The proof by the rupona method gives \[\left(4 - 1\right) \frac{20}{2} + 20\] = 200.
For the method of solution, the *regula falsi*, see Part I. Sec. 71 and 72, and for the *rupona method* see Sec. 73. The whole section is dealt with in Sec. 87, and the use of the symbol for nought in Sec. 60. (Kaye)

ii. The *sutra* beings. "Put what number you please in the empty place (or for the nought)." This is quoted on folio 23 recto and so is *tada vargam tu karayet*, etc.

**F 2**

*F 22, 23 recto.* The example may be represented by \( x + 2T_1 + 3T_2 + 4T_3 = 132 \), where \( T_1, T_2, \text{etc.} \) represent the values of the first, second etc. terms. Make \( x = 1 \), then the terms are \( 1 + 2 + 6 + 24 = 33 \) and the proper value of \( x \) is \( \frac{132}{33} = 4 \) and the series becomes \( 4 + 8 + 24 + 96 = 132 \).

All the technical terms here employed are of interest: *ichchha* an assumed number; *varga* a series; *prakshepa* something thrown in' or an interpolation'; *vartya* 'cancelled'; *dṛṣya* 'the given number'; etc.

ii. The term *karnika* (कार्निक) is practically synonymous with *ichchha* or *yadrichchha* 'that you please'. For an assumed number, Bhaskara uses *ishta* much in the same way. A good deal of the *sutra* is quoted on folio 2, 23 verso.

*23 verso.* i. The example may be represented by \( x = 2T_1 + 3(T_1 + T_2) + 4(T_1 + T_2 + T_3) = 300 \). Put \( x = 1 \), then the series becomes \( 1 + 2 + 9 + 48 = 60 \) and the proper value of \( x \) is \( \frac{300}{60} = 5 \) and we have \( T_1 = 5, T_2 = 10, T_3 = 45, T_4 = 240 \) and total \( T = 300 \).

ii. The example is solved on folio 24 recto.

**F 3**

*F 3. 24 recto.* The example may be represented by \( x (1 + 1\frac{1}{2}) + 2T_1 + 2\frac{1}{2}x + 3T_2 + 3\frac{1}{2}x + 4T_3 + 4\frac{1}{2}x = 144\frac{1}{2} \): Set \( x = 1 \) and the series becomes:

\[
\frac{5}{2} + \frac{15}{2} + \frac{52}{2} + \frac{217}{2} = \frac{289}{2} = 144 \frac{1}{2}
\]

which is the same as the given sum and therefore \( x = 1 \) is correct.
Problems

24 verso. i. The example may be represented by

\[ x (1 + \frac{1}{2}) + 2T_1 + \frac{5}{2}x + 3 (T_1 + T_2) + \frac{3}{2}x + 4 (T_1 + T_2 + T_3) + \frac{5}{2}x = 222. \]

Set \( x = 1 \) and the series becomes

\[ \frac{5}{2} + \frac{15}{2} + \frac{67}{2} + \frac{357}{2} = 222 \]

The same quotation shunya sthane... rupam datva occurs on folio 25 verso. See also at the bottom of folio 26 recto.

F 4.

25 verso. An example, of which only the solution remains.

ii. The example of which the solution is given on folio 26 recto.

F 5.

F 5. 29 recto. i. This is the solution of the example given at the bottom of folio 24 verso. Let \( x = 1 \), then the series becomes

\[ \frac{5}{2} + \frac{5}{2} + \frac{5}{2} + \frac{1}{2} = \frac{9}{2} \]

and the correct value of \( x \) is \( \frac{9}{2} + \frac{9}{2} = 1 \).

G 1.

G 1. 10 verso. i. Gives further proofs of the example on the obverse, namely:

\[ x_1 (1 - \frac{1}{3}) (1 - \frac{1}{4}) (1 - \frac{1}{5}) = 32, \text{ hence } x_1 = 108; \]

then two proportions in words and figures \( \frac{2}{8} : 1 :: 108 : 32 \) and \( a : 3\frac{a}{8} :: 32 : 108. \)

ii. Example—Of iron or ce refined three-tenths is lost. What is the remainder of twice seventy, tell me, O' Pandit? The loss on unity is \( \frac{3}{10} \) and the remainder is \( \frac{7}{10} \). The original quantity is 140 and \( \frac{3}{10} \) of 140 = 98. The loss is therefore, 42 and 98 + 42 = 140.

Proof. \( \frac{7}{10} : 1 :: 98 : 140 \)

Continued on fol. ii. recto.

iii. The example may be rendered:
The third part of the burnt bronze in three instalments (is lost). The amount given was one-hundred and eight. State the remainder, O’ Pandit.

The solution according to the rule gives

\[ 108 \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{3}\right) = 32. \]

But proceeding by steps \( \frac{1}{3} = 36 \) and the remainder is 72; \( \frac{2}{3} = 24 \) and the remainder is 48; \( \frac{3}{3} = 16 \) and the remainder is 32.

The proof may be represented by

\[ x_1 = \frac{32}{\left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{3}\right)} \]

G. 2

ii. Example—In purchasing one and a half palas the loss is one-third. State what would be the loss on eighteen. Since \( \frac{1}{3} = 9 \), the loss on unity, the remainder is \( \frac{2}{3} \). Now \( \frac{3}{3} \) of 18 = 14 and the loss is 4.

Proof by the rule of three : \( \frac{2}{3} : \frac{1}{3} :: 18 : 4 \) and \( \frac{1}{3} : 1 \frac{1}{3} :: 4 : 18 \)

Example—in refining bronze there is a loss of one-fourth. What would be the loss on 500 suvarnas four times refined?

The solution is lost. It amounted to

\[ 500 \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) = 158 \frac{13}{64} \text{ su\textsuperscript{o}} \]

\[ = 158 \text{ su\textsuperscript{o}} + 1 \frac{1}{64} \text{ to\textsuperscript{o}} \]

(Since 5 tolas = 1 Suvarna)

(11 verso.) This appears to have contained five proofs of the example on the obverse, for the present third proof is designated ‘the fourth’. The proofs are—

i. Missing.

ii. \( x^1 \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) = 158 \text{ su\textsuperscript{o}} + 1 \frac{1}{64} \text{ to\textsuperscript{o}} \),

therefore, \( x^1 = 500 \).

iii. \( 500 \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) = x^1 \) and \( x = 158 \text{ su\textsuperscript{o}} + 1 \frac{1}{64} \text{ to\textsuperscript{o}} \)

iv. \( x^1 = (158 \text{ su\textsuperscript{o}} + 1 \frac{1}{64} \text{ to\textsuperscript{o}}) / \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \) and \( x^1 = 500 \).
v. The first basis is \( \frac{500}{4} \text{ su}^o = 125 \text{ su}^o \) and remainder is \( 375 \text{ su}^o \). The second loss is \( \frac{375}{4} \text{ su}^o = 93 \text{ su}^o + 3 \text{ to}^o + 9 \text{ masha} \) (since \( 12 \text{ ma}^o = 1 \text{ to}^o \)) and remainder is \( 281\frac{1}{2} \text{ su}^o \). The third loss is \( 281\frac{37}{64} \text{ su}^o \) and the remainder is \( 210\frac{13}{64} \). The fourth loss is \( 210\frac{37}{64} \text{ su}^o \).

G. 3.

The example may be conjecturally restored: A traveller goes on a journey of 4 gavyutis and takes with him 4 prasthas of wine. After each gavyuti he drinks 1 prastha and then fills up his bottle with water. How much wine and how much water will there be at the end of his journey?

G. 3. (12 verso). The solution of the example on the obverse is now done by steps. The original amount of 4 prasthas is expressed in kudavas, namely 16.

<table>
<thead>
<tr>
<th>Kudavas of wine drunk</th>
<th>Kudavas of wine left</th>
<th>Kudavas of water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Originally</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>After first gavyuti</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>After second gavyuti</td>
<td>( \frac{1}{4} \times 3 )</td>
<td>9</td>
</tr>
<tr>
<td>After third gavyuti</td>
<td>( \frac{8}{9} = 2\frac{1}{3} )</td>
<td>6\frac{1}{2}</td>
</tr>
<tr>
<td>After fourth gavyuti</td>
<td>( 2\frac{7}{9} \times \frac{1}{2} = \frac{5}{4} \frac{1}{6} )</td>
<td>5\frac{1}{6}</td>
</tr>
</tbody>
</table>

G 3. 12 verso and 13 recto. Kumkum was purchased for 8, and at each toll office the toll was paid one-fourth. There were four toll-offices, calculate the total toll paid, and the Kumkum left, O' Pandit.

Solution: i. \( 8 \times \frac{3}{8} = 6 \), and 2 is paid as the first toll.

ii. \( 6 (6 - \frac{1}{4}) = 4\frac{1}{2} \), and the second toll is \( 1\frac{1}{2} \).

iii. \( 4\frac{1}{2} (1 - \frac{1}{4}) = \frac{9}{8} = 3\frac{1}{8} \), and the third toll is \( 1\frac{3}{8} \).

iv. \( 3\frac{1}{8} (1 - \frac{1}{4}) = \frac{5}{8} = 2\frac{1}{8} \), and the fourth toll is \( \frac{5}{8} \).
The total toll paid is:
\[= 2 + 1\frac{1}{2} + 1\frac{1}{6} + 1\frac{1}{3} = \frac{3}{2} + (64 + 48 + 36 + 27) = \frac{570}{32} = 537.\]

\[8 - \frac{15}{32} = \frac{81}{32} = 2\frac{17}{32}.

G 4.

G 4. (13 recto). i. Here are four ‘proofs’ of the example given on folio 12 verso.

(a) \[8(1 - \frac{1}{4}) (1 - \frac{1}{4}) (1 - \frac{1}{4}) (1 - \frac{1}{4}) = \frac{8}{3}^4.\]

(b) \[8\cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{8}{3}^4.

(c) \[8(1 - \frac{1}{4}) (1 - \frac{3}{4}) (1 - \frac{4}{4}) (1 - \frac{4}{4}) = \frac{8}{3}^4.

(d) \[x_1 = \frac{81/32}{(1 - \frac{1}{4}) (1 - \frac{1}{4}) (1 - \frac{1}{4}) (1 - \frac{1}{4})}, \text{ whence } x_1 = 8.

ii. There is a quantity of molasses weighing eigut bharakas. What will be left after giving away one-third, one-sixth and one-fifth.

\[8(1 - \frac{1}{3}) (1 - \frac{1}{6}) (1 - \frac{1}{5}) = \frac{8}{3}^4 = 3\frac{1}{2}.

iii. Example—By a gain of five-fourth, ten dronas are obtained. Let it be said. O best calculators, what will be the gain by three transactions.

Here the term labha seems to have meaning “capital profit”, what is termed the ‘mixed quantity’ misraka on folio 62.

(b) \[x_1 (1 - \frac{1}{4}) (1 - \frac{1}{4}) (1 - \frac{1}{4}) = 19\frac{1}{8} \text{ dro}^0 = 19 \text{ dro}^0 + 2a^0 + 0 \text{ pra}^0 + 2 \text{ ku}^0 \text{ whence } x_1 = 10.

The conversion table of capacity measures is given in folio 13 as follows (marked with asteriks):

4 adhakas (a^0) = 1 drona (dro^0)
4 prasthas (pra^0) = 1 adhaka
4 kudava (ku^0) = 1 prastha.

ii. Example—The capital of a certain banker is sixty. One half of it goes in loss and then he gains by one-third; next he loses
Examples

one-fourth of it and finally gains one-fifth; so that he has two gains. What is his gain and what is his loss and what the remainder and let that be stated.

Solution : \(60 \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \left(1 + \frac{1}{6}\right) \left(1 + \frac{1}{5}\right) = 36\).

Proofs : (a) \(x^2 = \frac{36}{\left(1 - \frac{1}{3}\right) \left(1 + \frac{1}{6}\right) \left(1 + \frac{1}{5}\right)}, \) whence \(x^2 = 60\).

(b) \(60 \left(1 - \frac{1}{3}\right) \left(1 + \frac{1}{6}\right) \left(1 - \frac{1}{5}\right) \left(1 + \frac{1}{5}\right) = 36\).

(c) \(x^2 \left(1 - \frac{1}{3}\right) \left(1 + \frac{1}{6}\right) \left(1 - \frac{1}{5}\right) \left(1 + \frac{1}{5}\right) = 36\).

whence \(x^2 = 60\).

G—5

G 5. ii. Example—A known amount of molasses equal to four is increased by one-third, one-fourth, one-fifth, one-sixth and then forty is lost.

No solution is preserved.

G 5. (14 verso). i. Example—An unknown quantity of lapis-lazuli loses one-third, one-fourth, and one-fifth, and the remainder after the three-fold operation on the original quantity is twenty seven. State what the total was, O wise one, and also tell me the loss.

Solution : \(\frac{3}{8}, \frac{2}{3}, \frac{1}{4}, \frac{45}{3}, 27 \div \frac{3}{8} = 45\) and \(45 - 27 = 18\) and this is the loss.

The meaning of ambha-loha, lapis-lazuli, was suggested by Dr. Hoernle.

ii. Example—Of the loss of iron the third is one-fifth of a masha. The original quantity is not known and neither is the remainder given but only the original remainder which quantity stands as twenty. Tell me what is the unknown, original quantity and what is the remainder.

The interpretation, however, is by no means certain. The solution is lost.
ii. Example—Of a ghataka of honey two-third is given, to the second two-fifth; to the third two-sevenths; to the fourth two-ninths, till only three palas are left). O' Pandit, state how much altogether was taken away by the tax collector.

H-1

H 1. ii. Rule : Having multiplied the parts of gold with the ksaya let this sum be divided by the sum of the parts of gold. The result is the average ksaya. This means:

\[ f = \frac{f_1g_1 + f_2g_2 + \ldots + f_ng_n}{g_1 + g_2 + \ldots + g_n} \]

where \( f \) denotes ksaya and \( g \) gold.

iii. Example—\( f_1 = 1, f_2 = 2, f_3 = 3, f_4 = 4, \) and \( g_1 = 1, g_2 = 2, g_3 = 3, g_4 = 4, \) therefore,

\[ f = \frac{1.1 + 2.2 + 3.3 + 4.4}{1 + 2 + 3 + 4} = \frac{30}{10} = 3. \]

H 2

H 2. ii. Example—Gold one, two, three, four; abandoned, the following mashakas one-half, one-third, one-fourth, and one-fifth.

\[ F = \frac{\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}}{1 + 2 + 3 + 4} = \frac{163}{60} + 10 = \frac{163}{60}. \]

Proof by the rule of three \( g : f_g : : g_r : g_rF. \)

H 3.

H 3. ii. Example—Mashakas of one and two, gold of two and five, mashkas of three and gold unknown. All that is known is the sum of mashakas six; and the average mashakas four. State the mashaka of the unknown gold.

Statement : \( f_1 = 1, f_2 = 2, f_3 = 3; g_1 = 2, g_2 = 5, g_3 = x; F = 4 \)
Problems

Solution: \[ \frac{4.6 - (2.1 + 5.2)}{3} = \frac{24 - 2 - 10}{3} = \frac{24 - 12}{3} = \frac{12}{3} = 4 \]

J 1.

J 1. Example—If a man requires six drammas for his livelihood for 30 days, for how many days will 70 men (guards of a fort) live on six drammas? The details are, however, uncertain.

K 1.

K 1. i. Example—What number with five added is a square and that same number with seven subtracted also being a square? What is that number? is the question.

Statement: \( x + 5 = s^2 \), \( x - 7 = t^2 \)

Solution: \( x = \left[ \frac{1}{2} \left( \frac{5 + 7}{2} - 2 \right) \right]^2 + 7 = 11 \).; by steps thus:

Having combined the added and subtracted number \( 5 + 7 = 12 \): that halved \( = 6 \); two subtracted \( 4 \); halved \( 2 \); squared \( 4 \); then that subtractive number \( 7 \) is to be added and by the addition of this \( 4 + 7 = 11 \); and this is the required quantity.

Proof: \( 11 + 5 = 4^2 \), \( 11 - 7 = 2^2 \)

For such problems, if \( x + a = s^2 \) and \( x - b = t^2 \), then the solution is,

\( x = \left[ \frac{1}{2} \left( \frac{a + b}{c} - c \right) \right]^2 + b. \)

L 1.

L 1. iii. Example—In two days one earns five; in three days he consumes nine. His store is thirty. In what time will his earnings be consumed?

Solution: \( t = \frac{30}{3 - \frac{9}{2}} = 60 \) and the amount earned in this time is \( \frac{5}{2} \) of \( 60 = 150 \) dinaras.

iii. Example—In three days one Pandit earns a wage of five and a second wise man earns six (\( \text{rans}a \)) in five days. The second is given by the first seven from his store and by this giving their possessions become equal. Let it be stated in what time,
Solution: \( t = \frac{2 \times 7}{\frac{5}{3} - \frac{6}{5}} = 30 \).

L 2.

Profit and loss.

The author gives a number of solved examples on problems concerning profit and loss. The following rules are given with illustrative examples:

[Let \( C \) be the capital, \( p \) the profit, \( M = C + p \), \( N = C \), the number of articles, \( C \) the cost rate, and \( s \) the sale rate, \( C = \frac{a}{b} \), where \( a \) is the number purchased for \( b \) drammas (but money measures are not generally stated), and \( s = \frac{C}{d} \), where \( C \) is sold for \( d \) money.]

(i) \( C = \frac{p}{\frac{c}{s} - 1} \);
(ii) \( p = \frac{Cc}{s} - C \);
(iii) \( C = \frac{M}{\frac{c}{s}} = \frac{C + p}{\frac{c}{s}} \);
(iv) \( C = \frac{-p}{1 - \frac{c}{s}} \).

L 2. ii. Example—Two Rajputs are the servants of a king. The wages of one are two and one-sixth a day, of the second one and one-half. The first gives to the second ten dinaras. Calculate and tell me quickly in what time there will be equality. (Indian Antiquary), 1881, p. 44).

Statement: \( \frac{1}{6}, \frac{1}{2} \), given 10.

Solution: The difference of the daily earnings.

\[ \frac{1}{6} - \frac{1}{2} = \frac{2}{3} \text{ dinaras.} \]

Therefore, a difference of 20 dinaras would be in 30 days.

L 2. 6 verso. i. Proof of example on the obverse—

\[ 1 : \frac{1}{6} :: 30 : 65. \]

\[ 1 : \frac{3}{5} :: 30 : 45 \text{ and } 65 - 10 = 45 + 10. \]

(ii. The rule means, \( C = \frac{p}{\frac{c}{s} - 1} \), where \( C \) is the capital, \( p \) the profit, \( C \) is cost rate and \( s \) is sale rate.
Problems

iii. Example—One buys 7 for 2 and sells 6 for three and finds his profit. What was his capital?

Solution: \( C = \frac{18}{7} + \frac{6}{3} - 1 = 24 \). The proof is given on folio 62 recto.

iii. Example—Eight articles are obtained for three and six are sold for four. The sum of the capital and profit is one-hundred and sixty. State, O’ best of calculators, what was the capital and what is the profit.

The solution is lost except for the first quotation, but part of a proof is given on folio 63, recto. The solution was,

\[
C = \frac{160}{16} - \frac{9}{9} = 90.
\]

Since \( C = \frac{p}{8/3} = \frac{6}{4} - 1 = \frac{16}{9} - 1 = \frac{7}{9} \)

\[ p = \frac{7}{9} C \] and \( C + p = 160 \), and the number of articles bought was \( \frac{3}{5} \) of 90 = 240.

iii. Example—With five four-squared are obtained by some man. For one-six are sold and fifty-six is the loss. Calculating purchase and sale let his capital be stated.

The solution is \( C = \frac{56}{1 - \frac{16}{30}} = 120 \) and the number of articles is \( \frac{5}{8} \) of 120 = 384.

L 4. recto. i. Proof of example given on folio 62 verso. 8:3::240:90 and 6:4::240:160 and 160 = 90+70.

ii. The rule means \( C = \frac{l}{1-c/s} \), where \( l \) is the loss sustained,

i.e. having investigated the selling rate multiply with the purchase rate and having subtracted from unity divide and the capital is obtained.
L 4. 63 verso. Proof of example on the obverse:

s:1::384:120 then with the selling rate 6:1:384:64 and 120 - 64 = 56.

ii. Rule—That which is the tax on cloth is taken in cloth: the tax on a piece of cloth is one-twentieth part.

Some one sells three-hundred. On the pieces being brought to market, two pieces are taken by way of tax: ten is (?) the selling price. What is the value?

M 1.

M I. 20 verso. i. A mere fragment: 12,000 mudrikas...

ii. Example—A snake eighteen hastas long enters its hole at the rate of one-halfplus one-ninth; of that, minus one-twenty first part of an angula a day. In what time will it have completely entered its hole?

Solution: Since 24 angulas = 1 hasta, we have $\frac{1}{2} + \frac{1}{9}$ of $\frac{1}{2} - \frac{1}{21}$: $\frac{18}{24} \times 24 = x$, whence $x = \frac{180}{80} = 2$ years 4 months 10½ days.

M 2.

M 3. 38 recto. i. Example—The earning of dinaras is difficult but consuming them is easy. One gives one-half increased by ratio on one-half (six times) for food for the poor. What is the amount consumed in 108 days?

Solution: $1 : \frac{1}{2} : \frac{1}{2} : \frac{1}{2} : \frac{1}{2} : \frac{1}{2} :: 108 : 1 \text{ di}^0 + 8 \text{ dha}^0 + 1 \text{ am}^0$ i.e., $1:(\frac{1}{2})^6 :: 108 : 1 \text{ di}^0 + 8 \text{ dha} + 1 \text{ am}^0$ and 4 amasas$^0$ = 1 dhanaka$^0$ and 12 dhanaka$^0$ = 1 dinara$^0$.

ii. Example—(This is not understood, but appears to refer to the number of hairs on the skin of an animal).

M 3

M 3. i. A mutilated example about Ravana and Sita. When Sita had been carried up 30 yojanas into the air she dropped something to earth, which turned over 8 times in $1\frac{1}{8}$ dhanusha How many revolutions did it make before reaching the earth?

(1 yojana = 8000 dhanusha)
Problems

Solution: \((1\frac{1}{4} - 1\frac{1}{6})\) dha\(^o\) : 8 revolutions:

\[30 \times 8,000 \text{ dhanusha} :: 210, 181, 181, 181\] revolutions.

ii. Example - A snake which is 100 yojanas, 6 krosas, 3 hastas, and 5 angulas long sheds its skin at the rate of 1 angula in 2 days. In what time will it be free?

The solution is given (?) on the reverse.

30 verso. i. \(1 \text{ an}^o : 2 \text{ d}^o :: 100 \text{ yo}^o + 6 \text{ kr}^o + 3 \text{ ha}^o + 5 \text{ un}^o\)  

\[429,867 \text{ years 1 month and 4 years.}\]

or \(1 : 2 :: 77,376,077 \text{ an}^o : \frac{154752154}{360} \text{ years.}\)

ii. An example about some garment falling to the earth. The elements are uncertain. Compare with the problem on the obverse (1).

M 4. 36 recto. i. "This land measurement is completed" may refer to the fragmentary example at the bottom of folio 32 verso, but it is doubtful.

ii. The example appears to refer to heaps of salt. If one heap or quantity weighs 1,075 palas how much will 56 heaps weigh?

\[1 : 1075 :: 56 : 30 \text{ bha}^o + 200 \text{ pn}^o\]

\[\text{or } \frac{1075 \times 56}{200} = \frac{60200}{2000} \text{ bhara} = 30 \text{ bha}^o + 200 \text{ pa}^o.\]

iii. One-tenth of a cowry is given in eighty eight... of this one twentieth and one-hundredth...

36 verso. i. The statement means 3 yo\(^o\) : 1 day :: 5 yo\(^o\) :: 21, 333 years 4 months,

\[\text{or } 3 \text{ yo}^o : \frac{1}{580} \text{ years} :: 5 \times 4,608,000 \text{ yo}^o : 21.333 \text{ years 4 months.}\]

\[\frac{5 \times 4,608,000}{3 \times 360} = 21,333\frac{1}{3}.\]

ii. A boat goes one-half of a third of a yojana plus one-third less one quarter, \(\frac{1}{3} + \frac{1}{3} - \frac{1}{4}\), yojana in one-half of one-third of a
day, but then it is driven back by the wind one-half of one-fifth of a yojana, \( \frac{1}{2} \) of \( \frac{1}{5} \) yojana, in one eight of three days. In what time will it travel one hundred and eight yojana.

\[
\text{108} = \left( \frac{1}{3} \text{ of } \frac{1}{5} \right) \times \left( \frac{1}{8} \text{ of } 3 \right) \times \frac{3}{8} = 1 \text{ year 3 months } 12\frac{1}{7} \text{ days}
\]

The details of the question and the solution are not clear.

M 5.

M 5. 34 recto. i. The problem is; Eleven birds feed on \( \text{prasriti} \) (handful) of corn; how many can feed on 8 Kharis of corn? It ends ‘Say, O' friend, say what are the Khagas, O' Sundari.

If this be correct, the same Sundari, “beautiful one” is used in exactly the same way as Lilavati is used by Bhaskara.

The solution is 1 \( \text{pra}^0 : 11 \text{ Kha}^0 :: 8 \text{ khar}^0 : 63,360 \text{ khagas} \), which would make 720 \( \text{prasriti} \) 1 khari; but there are many elements of doubt and the application of esha bahu pramanam to this particular problem is not clear.

ii. By certain persons one kala plus one pada and one yava are given in gold daily at the shrine of Shulin. What would be the amount of the gift in five years, five months and fifteen days... I desire to know that.

Solution—1 day : 1 yo^0 + 1 ka^0 + 1 pa^0 :: 5 years, 5 months 15 days : x.

or 1 day : 30 pa^u :: 1,965 day ; \( \frac{1965 \times 30}{192 \times 25} \) tola

= 12 tola^o + 3 dha^o + 1\frac{1}{2} \text{ am}^o.

Sec Part I, Sec. III (Kaye for measures of weight)

ii. The problem is about a diamond weighing 1\frac{1}{8} \text{ mashaka}, and obtained for 7 55 satera.

The statement means 1\frac{1}{8} \text{ ku}^0 + \frac{1}{4} \text{ ma}^0, 55 \text{ sa}^0, and indicates that 128 \text{ ma}^0 = 1 \text{ ku}^0 = 1 and that 40 \text{ si}^0 = 1 \text{ ma}^0. \text{ See part III (Kaye).}
Problems

M 6.

M 6. 37 recto. i. The question may be roughly restored: the Sun (Surya) traverses 500,000,000 yojanas in a day. State with certainty the amount of the journey of the Sun (Divakara) in a ghatika.

60 ghatika = 1 day = 30 muhurta.

The statement means 30 mu° : 500,000,000 yo° : 1 gha° : 83,333, 333$ yo° and it indicates that 2 ghatika = 1 muhurta (=$\frac{1}{3}$ of a day). The origin of the length of the daily journey of the Sun, namely 500,000,000 yojanas, is not known.

ii. The chariot of the Sun (Bhanu) is surrounded by the groups of Gods, great snakes, Siddhas and Vidyadharas. In a day and night, its journey is said to be half a hundred kotis. Tell me, O beat of calculators, how much in one muhurta?

1 day = 30 mu°

30 mu° : 500,000,000 : : 2 gha° : 16,666,666$ yo°.

57 verso. i. The remnant of a problem possibly related to the daily motion of Jupiter, which according to the Surya Siddhanta amounted to very nearly 5 minutes of arc (lipta).

ii. If Bhanuja (Saturn) moves through a sign in two and a half years, state, O knower of the truth, what will its motion in a solar day be equal to (1 sign = 30° = 108(00")? The solution is 2$\frac{1}{2}$ years : 1 sign : : $d$ degree : $x$

$x = \frac{1 \text{ sign} \times 360^\circ}{2\frac{1}{2} \text{ years}}$

$= \frac{30 \times 60 \times 60 \times 2}{5 \times 360} :: \frac{108,000}{900} = 120'' = 2 \text{ minutes.}$

of arc (not 2 seconds as stated in the text, where vilipta appears to have been written by mistake for lipta). The terms employed are all orthodox except perhaps vasara for solar day, but its special use is quite intelligible.
M 7.

M 7. 47, recto. i. This appears to Partha, the Mahabharata hero who pierced each soldier with \(16 \left(1 - \frac{1}{3}\right)\left(1 + \frac{1}{3}\right)\) arrows and slew four divisions of the army. How many arrows did he use?

\[
1 \text{ si}^0 :: 16 \left(1 - \frac{1}{3}\right)\left(1 + \frac{1}{3}\right) :: 4 : x; \ 21, 870: 2, 624, 40
\]

The abbreviation \(\text{si}^0 = ?; \ \text{a}^0 = \text{anikini. See Part I. See 52 Kaye).}

There is a very similar example about Partha in the *Lilavati* which has already been quoted (Part I, 47 recto).

i. Apparently, 3 chamus=1 pritana; 3 pritanas=1 anikini and 10 anikinis=1 akshauhini. The statement means that a *Patti* consists of ratha+1 gaja+5 nara+3 turaga i.e., 1 elephant+5 foot soldiers+3 horsemen) and that an akshauhini contains 3,710 of each of these, namely—

\[
\begin{align*}
1 \times 3^7 & \text{ chariots} = 21,870 \text{ chariots (ratha).} \\
10 \times 3^7 & \text{ elephants} = 21,870 \text{ elephants (gaja).} \\
5 \times 10 \times 3^7 & \text{ footmen} = 109,350 \text{ footmen (nara).} \\
3 \times 10 \times 3^7 & \text{ horsemen} = 65,610 \text{ horsemen (haya).}
\end{align*}
\]

Total 218,700

Albiruni gives the following scheme:—

<table>
<thead>
<tr>
<th>Each</th>
<th>akshauhini</th>
<th>has</th>
<th>10 anikini</th>
</tr>
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<tr>
<td>&quot;</td>
<td>anikini</td>
<td>&quot;</td>
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<tr>
<td>&quot;</td>
<td>chamu</td>
<td>&quot;</td>
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</tr>
<tr>
<td>&quot;</td>
<td>pritana</td>
<td>&quot;</td>
<td>3 vahini.</td>
</tr>
<tr>
<td>&quot;</td>
<td>vahini</td>
<td>&quot;</td>
<td>3 guna.</td>
</tr>
<tr>
<td>&quot;</td>
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<td>&quot;</td>
<td>3 gulma.</td>
</tr>
<tr>
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</tr>
<tr>
<td>&quot;</td>
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<td>&quot;</td>
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</tr>
<tr>
<td>&quot;</td>
<td>patti</td>
<td>&quot;</td>
<td>1 ratha.</td>
</tr>
</tbody>
</table>
**Problems**

**M 10.**

*M 10. 55 verso.* If tola costs thirty-five drammas (dr°), what will be the price of one and a half tolas, one and a half mashakas and one and a half andikas and one and a half yavas.

1 tola = 12 mashakas; 1 mashaha = 4 andi = 16 yava.

\[ 1 \text{ tola}^0 + 1 \frac{1}{2} \text{ ma}^0 + 1 \frac{1}{2} \text{ an}^0 + 1 \frac{1}{2} \text{ ya}^0 = 319 \frac{1}{2} \text{ ya}^0. \]

Statement - (i) \( 1 \text{ tola}^0 : 35 \text{ dr}^0 :: 1 \frac{1}{2} \text{ tola}^0 + 1 \frac{1}{2} \text{ ma}^0 + 1 \frac{1}{2} \text{ an}^0 + 1 \frac{1}{2} \text{ ya}^0 : 58 \frac{76}{192} \text{ dr}^0 \)

or \( 1 : 35 :: 319 \frac{1}{2} : 192 : 58 \frac{76}{192} \text{ dr}^0 \)

**M 11.**

Example—One produces ten and a half in two and one-third days. For the sake of religion, he gives thirteen and one-third in three and one-eight days; he offers for Vasudeva one-quarter less than thirteen in eight and a half days. Desiring reward in a future world he gives i.e. \( 12 \frac{3}{8} \) to Brahmanas for food, one-third in three and one-fifth days... ... .. (two and a quarter in five days... ... (con’d. on M 12).

**M 12.**

*M 2. 43 recto.* and also twelve and a half in thirty-three and one-third days for the best wine for the consumption of merchants. In the treasure house is stored twelve hundred. Say, O Pandit, how long can this expenditure continue. The statement means—

Daily income = \( \frac{10 \frac{1}{6}}{2 \frac{1}{6}} = \frac{9}{2} \).

Daily expenditure = \( \frac{13 \frac{4}{6}}{3 \frac{4}{6}} + \frac{12 \frac{2}{6}}{3 \frac{2}{6}} + \frac{1 \frac{1}{6}}{1 \frac{1}{6}} + \frac{1}{3 \frac{1}{6}} + \frac{2 \frac{1}{6}}{5} + \frac{12 \frac{1}{6}}{33 \frac{1}{6}} \) = 1807

\( = 240 \)
The daily loss is, therefore, \( \frac{1807}{240} - \frac{9}{2} = \frac{727}{240} \) and

\[
\frac{727}{240} : 1 :: 1200 : x
\]

where \( x \) the period.

\[
x = \frac{1200 \times 1}{\frac{727}{360} \times 240} = \frac{800}{727} \text{ years.}
\]

M 14.

50 recto. At the top of this page is the remnant of a problem. too broken up to make out. The rest of the page is devoted to what appears to be a colophon. This is not all clear but what remains seems to state that the work was written by a certain Brahman, a prince of calculators, the son of Chhajaka. It also refers to the importance of the science of calculation which, it is said, we owe to Shiva.
The Bakhshali Text
Transliterated in Devanagari Script
| A1          | 40<sup>a</sup>r, 39<sup>a</sup>r, 38<sup>r</sup>b, 40<sup>a</sup>v, 39<sup>a</sup>v, 38<sup>v</sup>b | E4          | 21<sup>r</sup>, 21<sup>v</sup> |
| A2          | 39<sup>r</sup>, 40<sup>r</sup>, 38<sup>r</sup>, 39<sup>v</sup>, 40<sup>v</sup>, 38<sup>v</sup>. | E5          | 22<sup>r</sup> |
| A3          | 40<sup>r</sup>, 39<sup>r</sup>, 38<sup>r</sup>, 40<sup>v</sup>, 39<sup>v</sup>, 38<sup>v</sup>. | F1          | 22<sup>v</sup> |
| A4          | 54<sup>v</sup>, 54<sup>r</sup> | F2          | 23<sup>r</sup>, 23<sup>v</sup> |
| A5          | 35<sup>r</sup>, 35<sup>v</sup> | F3          | 24<sup>r</sup>, 24<sup>v</sup> |
| A6          | 51<sup>r</sup>, 51<sup>v</sup>, 35<sup>v</sup> | F4          | 25<sup>r</sup>, 25<sup>v</sup> |
| A7          | 51<sup>r</sup>, 51<sup>v</sup> | F5          | 26<sup>r</sup> |
| A8          | 52<sup>r</sup>, 52<sup>v</sup> | F6          | 26<sup>v</sup> |
| A9          | 29<sup>r</sup>, 29<sup>v</sup> | G1          | 10<sup>r</sup>, 10<sup>v</sup> |
| A10         | 27<sup>r</sup>, 29<sup>v</sup> | G2          | 11<sup>r</sup>, 11<sup>v</sup> |
| A11         | 1<sup>r</sup>, 1<sup>v</sup> | G3          | 12<sup>r</sup>, 12<sup>v</sup> |
| A12         | 2<sup>r</sup>, 2<sup>v</sup> | G4          | 13<sup>r</sup>, 13<sup>v</sup> |
| A13         | 3<sup>r</sup>, 3<sup>v</sup> | G5          | 14<sup>r</sup>, 14<sup>v</sup> |
| B1          | 8<sup>r</sup>, 8<sup>v</sup> | G6          | 15<sup>r</sup>, 15<sup>v</sup> |
| B2          | 9<sup>r</sup>, 9<sup>v</sup> | H1          | 16<sup>r</sup> |
| B3          | 7<sup>v</sup> | H2          | 16<sup>v</sup> |
| B4          | 4<sup>r</sup>, 4<sup>v</sup> | H3          | 17<sup>r</sup>, 17<sup>v</sup> |
| C1          | 5<sup>r</sup>, 5<sup>v</sup> | J1          | 18<sup>r</sup>, 18<sup>v</sup> |
| C2          | 6<sup>r</sup>, 6<sup>v</sup> | J2          | 19<sup>r</sup>, 19<sup>v</sup> |
| C3          | 7<sup>r</sup> | J3          | 30<sup>r</sup>, 30<sup>v</sup> |
| C4          | 65<sup>r</sup> | K            | 65<sup>r</sup> |
| C5          | 56<sup>r</sup>, 56<sup>v</sup> | L1          | 60<sup>r</sup>, 60<sup>v</sup> |
| C6          | 64<sup>r</sup> | L2          | 61<sup>r</sup>, 61<sup>v</sup> |
| C7          | 57<sup>r</sup>, 57<sup>v</sup> | L3          | 62<sup>r</sup>, 62<sup>v</sup> |
| C8          | 45<sup>r</sup>, 45<sup>v</sup> | L4          | 63<sup>r</sup>, 63<sup>v</sup> |
| C9          | 46<sup>r</sup> | M1          | 20<sup>r</sup>, 20<sup>v</sup> |
| D1          | 46<sup>v</sup> | M2          | 33<sup>r</sup>, 33<sup>v</sup> |
| D2          | 70 (a, b, c)<sup>r</sup>, 70 (a, b, c)<sup>v</sup> | M3          | 32<sup>r</sup>, 32<sup>v</sup> |
| D3          | 69<sup>r</sup>, 69<sup>v</sup> | M4          | 36<sup>r</sup>, 36<sup>v</sup> |
| D4          | 68<sup>r</sup> | M5          | 34<sup>r</sup>, 34<sup>v</sup> |
| D5          | 31<sup>r</sup> | M6          | 37<sup>r</sup>, 37<sup>v</sup> |
| D6          | 67<sup>r</sup>, 67<sup>v</sup> | M7          | 47<sup>v</sup>, 47<sup>r</sup> |
| D7          | 28<sup>r</sup> | M8          | 48<sup>r</sup>, 48<sup>v</sup> |
| E1          | 66<sup>r</sup>, 66<sup>v</sup> | M9          | 49<sup>r</sup>, 49<sup>v</sup> |
| E2          | 53<sup>r</sup>, 53<sup>v</sup> | M10         | 55<sup>r</sup>, 55<sup>v</sup> |
| E3          | 58<sup>r</sup>, 58<sup>v</sup> | M11         | 44<sup>v</sup>, 44<sup>r</sup> |
|             |                 | M12         | 43<sup>r</sup>, 43<sup>v</sup> |
|             |                 | M13         | 42<sup>r</sup>, 42<sup>v</sup> |
|             |                 | M14         | 50<sup>v</sup>, 50<sup>r</sup> |
A 1.

(40a recto, 39a recto, 38b recto, 39b recto 40b verso, 39a verso 38b verso)

यज्ञ य. ग...भाग्य चैव कार्येत् कृत्यंगुल्य...पुष्ट शल-इयं चैवोचरे शालग-कते: वैपुल्याधि...श छादन नृशकस्त्रथा। सतंपत्रच भवेत् चां भक्ति रथभें...र....

धा सत्य पञ्चनाग्नितिः स्व-विशेषं श्रेयस्य तस्य भवेत् श्रेयस्य किं क...तथा गम।

क्षेत्रस्य स्थापनः क्रियते।

क्षेत्र १००

...१५ | १२ | १३ | ७ | ६ | करणम्। क्षेत्र......३०० दं घ ...

...चैव तत्त्व फल...गुणिता जाता | ६२६० | एप वाहस्य काण्ड प्रमाणे

...शेषे मूल्यं करतवः.

अन्धवेष्ठिः चतुष्योक्तिः ल। सुपुष्पि विशाले मण्डलीकृ दलिका एषांक्षेत्रं शालिकते

...यथे तदान्तरं कार्यं। गुणविष्ट्य क्रिया उदाहरणं तदस्य...मक तत्त्व शालिक दशाधीकानां किं मूल्यं: तद्वते अः

| १ ...२३ | मूल्यं | ६२६० मण्डले आ हि | फ... | १

A 2.

(39 recto, 40 recto, 38 recto, 39 verso, 40 verso, 38 verso)

...क्ष...दशा चतुर्दशा तृतीयस्य चतुर्थम्...भाग्यस्त्रयें पञ्चम्...भाग किषांक्षेप दशगुणा। सप्तम क्ष...ज यं शालम्। सर्वं भिषापि दृष्टं च शालनि......

...धानं १......१०। एप एकक्ष भागगुणिता जा ... ६०।१४०।२००।३०० एवं धानं ९२०० प्रत्यय तृतीयिकेन...३०...१२००

| १५४ | १५६ |
| १६० |
| १६० |
| २०० |
A 3.

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स्थाप्य

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</tbody>
</table>

कृष प्रशेषक तत्त्व प्रक

क्रियते ॥ चतुर्भिन्नविनिर्विषधिः ... प्रथम राशि योग । 90 । वर्धम

मधु घट...हितय पक्षाय योग । 75 । वर्धम । 5 । पानीमु ॥ तृतीय पक्षय क्रियते

योगं । 80 । वर्धम जातं । 6 । पयसा...हितय...गुणेतु ॥ एको...कृतं शतचरमं

पञ्चमिः: पुरुषवत्त्वं किमां प्रथम धनमु ॥

...920 ॥ 2247...पमु...तु केवल कोष । 16 अनेनात्र भाग

<table>
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<th>32</th>
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जाता...१४१५२१२८। एवं ६०
(54 verso, 54 recto)

(i) """"$\text{ष्टिरैणं} च ल्यून च तुल्यस्य धनं भवेत् । ल्यूनं ।""""$\text{संमुन्तम्} ।
एकार्तितिभि । कृतो दीनारेत्र । आयुङ्द्याधुष्यथः वदः।$
करणम् । यस्य पारं न जातेः । एतद् प्रथमस्य धनम् ।

(ii) """"$\text{च दत्तवान् हस्ता} ।$\text{वेषाणं । ० । २ । २+ ।}""""$

<table>
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</tbody>
</table>

याताः । तत्त्वोऽमवियो । कृताः राशयः ।

(55 verso, 55 recto)

""""$\text{वर्तमानः धर्मः} । \text{वर्तमानः} ।$

A 5

(35 recto, 35 verso)

""""$\text{कस्मात् कारणः} ।$\text{तत्त्वोऽतिविग्न्यस्यवियोगम्} ।$\text{भाजितः} ।$\text{पुष्पः} २६ \text{अग्निः} ।$

<table>
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""""$\text{सुलेन} । एत दिगुणं । दिगुणं । यस्य धनम् ।$

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
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</table>

तदेव सार्धः । अस्यार्धः । १ । युर्त्वम्यास ।
A 6.

(51 recto, 51 verso, 35 verso)

भाजितो हितवा । तत्रोत्तर ९ १ १ । युत् । २ । १ ३ ३ । ६ । ... । ४ २ १ ।

एवाः ईश्वर भाजिता । पुर्वः । १ ३ ३ । एवां सदृशे । ४ । धनः । ९ ६ । अनेन ।

गुणित जातं । ४ । एव प्रथमस्थ धनः ... दिग्नानं । १ २ । दिग्नानं । १ ४ । एते ।

द्वितीयस्य ... गुणं । २ १ । दिग्नानं । ४ २ । ध्वसं । ३ ६ । एते । न्यासः प्रत्यः ... ।

दशमप्र-वृन्दानां चतुर्दशं एकौनचलवारिणः। तत् ... पादार्थ विभागः । ४ । । 

| एवं दीर्घं । १ । एव प्रथम एति ... ।

उदा ... । ६ । ... यांग ९९१ शेषा । पुर्वः ।

भाजिता पुर्वः । १ । १ । १ । एवां सदृशे । युतः क्रत्वा । ३ ६ । भाजिता । ६ । ।

एव परशुराम महिमार्य प्रस्तूक शालेपुर भाग ... 

| १ शाला । १८० ग्रामाः । १ । फलं ४६ । 
| १ शाला । १८० महिला । १ । फलं १६ । 

A, 7

(51 recto, 51 verso)

| १ | १ | १ | १ | १ | १ | १ |

| १ | १ | १ | १ | १ | १ | १ |

... २ । ३ । एवां युतः । ६ ... । ४५ । शेषा । पुर्वः स ४ ... ।

अनेन भाजितार्यः यथा भवति । १ २ । १३ । १ ४ । १५ । एकं । ४४ ... ।

(२) उदा ... कष्टिधरः राजा वदेद दानं सप्तपश्चातशनम् गुहोः।
करणम्। उत्तरः तदोत्तर राशिनां योग ६७ एव धना दूषया शोधनीया जाता २४२**। पुरुष । १। ३। ६। २७। ५९। योग ९२९ अनेन** जाता । २। एष हौ प्रथमस्य धनम्।

| २ | १५ | ४८ | १४७ | ४४४। एष्या... |
| १ | २ | २ | २ | २ |

A ८।

(५२ recto., ५२ verso.)

(i) **५७*** त्वेदं जातं ३ अनेन चालिण गुणयं जातं १२०***स्वर्णशुराणाम्***

| प्रत्यय तैराशिकेन | १ | १२० | १ | १२२ | ... |
| | १ | १ | १० | १ | ... |
| १६ | १२० | १ | १५ | ... |
| वर्ष | १२० | १ | १५ | ... |
| १६ | १२० | १ | ३० | ... |

(ii) उदा०।। धना स्वमयं संशोध्यं***चतुरीयकम्।
तत् शेषम् पञ्चचमो भागो***शालदयम्।
अशीत्याधिकं धनं चैव किमाण् प्रथमं धनम् ॥

अस्य द्वारां शतानां पादं धर्मम् ॥

शतं भवितं १५० भाद्रापि पञ्चं भाग ३० ॥ एवं... |
| १ | १ | १ | पञ्चपिण्डुः २५० ॥ |
| २ | २ | २ | २ | २ | २ |

पञ्चमी जाती करणं कुलं २५० ॥ अष्टशुद्धि २६ शतं ४० धनु २५० ४० २६

गुणितं जातं ४०० एष फलं भवितं ॥

A 9.

(२९ recto. २९ verso)

... \( \text{विनिदिशेतुं} \) ॥

उचाहारं ॥ धना... \(^{\text{...विनिदिशेतुं}} \) ॥

स्याधिकं योनिमं धनं तत्र ब्रमोदशः
हितियाधिकं योनिमं स्वयं
अधि द्वादशयोनिमं धनं पञ्चचदश धमः
एकाक्षरं धनं...विचित्रे कष्टांममः

| १३ | १४ | १५ |
| १ | १ | १ |

...प्रथमपद्यम् तत्रैष्ठा पञ्चं ॥ ५ ॥ ततू प्रथम... \(^{\text{...विचित्रे कष्टांममः}} \)

... १३ / १४ / १५ ततदीशोद्धौषधेक्रमतादि...एत \(^{\text{...विचित्रे कष्टांममः}} \)

चतुर्दशिष्ठि शोष्य केषं ६ एतूऽ पञ्चं

... \(^{\text{...हितियाधिकं योनिमं धनम्}} \)
वित्तित्य तूर्य योग्मत्रं धनं सफलश्च स्मृतः
तृतीयाधारुवर्यो...

चतुः पञ्चक सर्पं तु धनं एकोक्तिसमाति।
प्रथम तत्त्र च
एकाक्त्य धनं कि स्पष्ट् वेचिछाँ...

| १६ | १७ | १८ | १६ | २० |
| १ | १ | १ | १ | १ |

करणम्॥ इत्यादि दानीशोधवेत् क्रमात् तदादि १६ तुदः॥तृतीयायं श्लोक्या
७ चतुर्थायं श्लोक्यं १२ पञ्...

A 10.

(27 verso, 29 verso, 27 recto, 29 recto)

रस स्नेहम् एवामुनक्षे नूष्योत्कति...

| ५ प्रत | ६ दिन | १०धिल | ५ चो | ११ पञ |
| ७ दिन | १०धिल | ५ चो | १५पञ | ५ प्रत |

युतं जातं प्रत्यक्षः १६। १७। १५। १६। २०। एवं सर्वत्रं कार्येत्
करणम्। पृथ्वी रूपं विनिमितिः। पृथ्वी रूपं लिंगं जातं...म्यासो तत्त्र गुणं
| २ | ४ | अङ्गारं | १२। रुपहीं निम्न। अमागा चतुर्फः पञ्चम् ५५। अव धिरं
| ५ | ५ | अङ्गारं | १५। १६। एव लिंगम्...ता गुणं...विच चतुर्फः पञ्चम् ५ | ५ | ४ | एव...

थुबं॥ गुणे...क धनम्॥ गुणम्यासो रुपहीं लब्धं हुँ...

[४ = उपछानीय]

A 11

(1 recto, 1 verso)

(१) म्यासं... हुँ... चतुर्गुणं पञ्चगुणं हस्तवर्त्त धनं ज...पञ्चगुणं २५॥
| १ |}

नवम सूत्रं ५ ॥
(ii) सूत्रम्। गुणो गुणम् रुपमुत्री याचना गुति संगुणि:।
   गुणनेन:। गुणम्:। रुपमुत्री भास्तितो।
   विचारित याचना: किति गुणसाप्तोवखं विढ़ि:।
   एवं सूत्रम्। द्वितीय परेच विचारितसि।
   दशम सूत्रम् १०।}

(iii) सूत्रम्। द्वितीय विशोध्यंज्ञेद्ध-गुत्तत्त्त्तर्वतनम्।
   ....थ्यं तत्र प्रोक्तव धनाविश्व विनिदिशेत्।

(iv) उदाहरणं। पञ्चायानं विद्यामस्ये विक्रियाते किलं।
   तत्रक्रस्वा मणि-विक्रीता मणि-सूल्यं कियदृ्भेवत्।
   ....... दम्।
   अर्थ विनिभाग वादोऽं पञ्चभाग वादसि च।

......थ्यं तत्र प्रोक्तव: सदृशं क्रियते......जाता

| १२० | ५० | ६० | १५० | ३५० | ६० |
| ६० | ६० | ६० | ६० | ६० |

८०। ७५। ७२ एवं योग दृष्टे जाता ४३७ अतो:३७७ एव मणि सूल्यम्।
चतुर्णां संक सवर्षवम्।

प्रथमसत्त संकार्ब:७०। ८०। ७५। ७२ चतुर्णां योग ३७७ प्रथमाभि: बन्धितवर्त्यं ३७७ प्रथमसत्त धानं प्रथम धानं।

३४७ द्वितीय पञ्चभाग सवर्षव तत्र द्वितीय पञ्चभाग २० एव युतं ३७७ एव द्वितीयसत्त धानं भवति।

पुनः प्रथम द्वितीय चतुर्ण पञ्चभागः सवर्षव ३५७ तत्रितसत्त पारि २० एव युतं ३७७ एव तृतीयसत्त धानं भवति।

पुनर्वर्ष प्रथम-द्वितीय-तृतीय-पञ्चभाग ३६२ चतुर्वर्ष पञ्चोपाय १५ एव युतं ३७७ एव चतुर्वर्ष धानं भवति।
(१) स्य धनं भवति॥

अयः प्रथमः त्वमः शिष्टं शेषं ३७७॥

| अयः द्वितीयस्य | ६० | एवं ३७७ द्वितीयस्य भवति॥ |
| : | : | : |
| १२० | ३० | ८० | ६५ | ७४ | ७२ |

अयः तृतीयस्य क्रियते | ६० | एवं ३७७ तृतीयस्य धनं भवति॥
| : | : | : |
| १२० | ५० | २० | ७५ | ७२ |

चतुर्थस्य क्रियते | ६० | एवं ३७७ चतुर्थस्य धनं भवति।
| : | : | : |
| १२० | ४५ | १५ | ७२ |

पञ्चमस्य क्रियते। स्थापनं | ६० | एवं पञ्चमस्य ३७७।
| : | : | : |
| १२० | ५० | ७५ | ९२ |

एवं मणिपूल्यं प्रश्न॥

(२) उदार। अयोध्य विदित विभवं वर्णिक इयम्।

| द्रित् | दलं तथा……… |
| : | : | : |
| ७+ | ३+ | ५+ |
| १२ | १२ | ६ |
| १२ | १२ | ६ |
अंश विशिष्टक्षण विशालध्वेत्र प्रत्र श्वेतम्। एवं...क्रियते

| २५ | ४ | ११ | कुप्यातु। परिवर्तनम् | १२ | ४ | ६ | चेष्टे
| १२ | ४ | ६ |

...जातमस्य। | ५२४ | ५३७ | ७५५ | प्रोक्यम् जाता।
| ५२४ | ५३७ | ७५५ | एवं युत्त क्रियते...जाता। २५५७। चेष्टे
प्रोक्यम् १०५५ एतमणि मूलव्

A 13

(३ वर्षो, ३ रेक्टो)

उदातो।...बितियस्य ह्यातः उष्ट्रैदश तृतीयस्य...प्रदत्त च परस्परं
पूर्वाणम् तु वणिज्यो मूल्यं वा माणिनां मृत्युक यदि...वावं ततोमेचिष्ठि
संख्याः।

१ ए ५० | ५ ह० | ऊ १० |
| १ | १ | १ |

वणिज्यका ३ देयं...वणिक् पिण्ड हतं। पिण्ड ७५१०। देयं ३ मुख शेषः
४१६७ तत शेषं परस्पर कृत्त गुणित जातं。

१६६।१६६।१६६। स्वभेदन सु विषमं | १६४ | १६५ | १६६ |

लब्धं ४२१२८१२४। एक प्रत्येक मूल्यं एककृत्य...गुणिता जातानि अवशे
हृदी कृष्टिः। २५४।२५२।२५०। एककृत्य...जाता। ०६२।०६२।०६२।
एतेसमधना जा।

(१) स्वास्थसमधना जाता प्रस्तमूल्यं तदुवः ताम्

| ४ ए ५० | ५ गो० | ६ शात० |
| १ | १ | १ |

एवं प्रस्त मूर्त्यं गौ:। दल्लेसमधना जाता। १७१७१७......सम-
सतं १३
(ii) सूत्रम् II एकयुतानं संवय दिः...हीना च।
एवं ताबत्त कार्यं याबतुः पुर्वं समा भवति।
सन्त्र पतेभिनितिति स्थित
चतुर्दशं सूत्र १४

(iii) सूत्रम् II गतिस्येव विशेषणं विभिन्नं पूर्वं गतुः। तेनैव कार्यं भवति ।
केन तु।

(iv) उदाहरणं योजन गते शत................

B 1.
(८ recto, ८ verso)

(i) दिगुणं प्रभवं गृहं। दिगुणं निपटं तथा।
उत्तरेण भजेणां लघुं रूपं विनिविशिष्ट।

(ii) उदाहरणं II वर्तमानं भूतकं X कपिष्टल्लको दस्स मायकरम्।
यहाँ कुश्ते तत्र कस्म अद्विकमानवः
द्वितीयं किष्टे कस्म द्रुचादि गुरुश्चतरम्।
पवं तत्र तु भवति तेन कालेन सास्त्यात्मकं

आ २ | उ १ | पा ० | प्रतितः १० | १

दिगुणं प्रभवं गृहं। प्रभवं | २ | दिगुणं | ४ | नियतं पुनः हि...

१६ [उत्तरार्थं भाजयेत्] उत्तरम्

सूत्रम् II हृतोतिभवः गन्तव्यमती भाग। गन्तत् एकाकाशमवन्गम गुलासंगुण्ण

उदाहरणं II नियो रुवोस्वेद्वशमिर्युष्टे हुय पच्चकं। गन्तव्यं योजनशतं...

हृतोति (५)
भवेत् | हृ ११ | ह्यमणां रघुस्य | ५ | गन्तव्यं योजन १०० | १

हृतोतिभवः गन्तव्यं तत्र हुय | १० | गन्तव्यं भो | १०० | अक्ष भाग हुमें
लघुं | १० | तत्र युक्तावास | ७ | एते संगुण्णं परिष्कृतं जातं योजनायैवोषः
प्रत्येकं पञ्चचक्रस्त संगुण्णं जातं किष्टे। यदि व...योजनां पुष्पं...

* X = जिल्लासूत्रेषः, d = उपदमानीय
उदाहरणः...तस्माद द्विजावमित्।
तद्गुणस्ते समं भक्त्वा दशेः...समाप्तवान्।
सभस्यम् कृतिमात्रा कति विप्रा कृति प्रष्टम्।

आऽ १ | उऽ १ | प० ० | लखः १० |
      | १  | १  | १  |

करणम्। लब्धं हिरण्यतम कुलं तत्र लखः १० | हिरण्यं २० | तथ्यां १६ ।
उत्तरेण विभाजितमवोत्तरं । १ । अनेन भक्त्वा जातं तदेष त्वाद्विहिंक। १५ | अर्थ प्रश्ना
प्राप्तमा एकोनप्रश्नति।

स्वाप्नः...प्रत्ययं आऽ १ | उऽ १ | प० १५ | ह्योपलक्षणेन फलं १५
      | १  | १  | १  |

योऽ ६ | शयोऽ १ | योऽ ७० | गन्तव्यं
      | १  | १  | १  |

अलब्धं संयोग | ७ | विस्मयं | १ | गन्तव्यं ह्योपलक्षणं जातानु लखः । १ ।
 हिरण्यं २० | एशालस्यः। अर्थः...अर्थं कालोपेयं। अनेन कालेनप्रथमां
 ।...। रमेश स्वर्गिकस्य समागम। भवति।

तथथ वैशालिकेन प्रत्ययं। यथें क्ष्यं पद्योजना तदावशानां नं अर्थ
 १  | ६  | योऽ २० | फ० १२० |
      | १  | १  | १  | १  |

सत्तति शोकश्च शेष अत्स्पतिः । ७० | आगतप्रथचार | ५० | अर्थ...।
 १  | १  | १  | १  |
( ११५ )

B ३.

[७ वर्षों]

[i] ........

आदि विशेषदः आदि ३ नियत ६ विशेषदः ४ उत्तरार्थभाषितम्। उत्तरं ४ अनेन भाषितं २ जातं २ लघुस्कप । एष्य रूपाधिकं ३ एष्य कालं।

द्वितीयश्च वैरागिकेनः १ ७ ३ ७ १ १ २ २ २ २ ।

एष्यसमधनं जाता ॥

(ii) उदारं ॥ आदेक उत्तरवर्ण्यहितीय—पञ्चप्रत्ययम्।

केन कालेन समता वद मे गणकोत्तम ॥

आदि ३ ६ ४ ५ ० ० ० नियतनिन्तयं ५।

आदि विशेषदः ॥

B ४।

(४ रेक्टो ४ वर्षों)

(i) योजन पञ्चकम्। सप्तविनानि तस्येव गतस्य। परत्र विशीय नय

योजनेन गतकेन।

१ ७ ३ ७ १ ० ० ० ० ।

गतित्स्वयं विशेषशयवः ॥ गति ५ ६ ४ विशेषं ४। विशेषं । ४ ४ ४ ।

४ ४ ४ ।
उदाहरण ॥ अष्टादश योजना एकेन दिने याति ।
तस्याष्ट दिना गतस्य ।
द्वितीय यथा विल्के योजना दिने याति ।
केन कालेन सास्याताम ॥

eyamākāvyaṃ पद्यमविकित पूर्वार्धः ॥

पद्यदर्शन सूत्रम् १५ ||
(iv) सूत्रम् ॥ आधोविषेष कर्तव्यवस्थरस्य विशेषतः ।

विभक्तमुलते ।

उत्तरं ॥ २ ॥ विनाशम् ॥ १ ॥ आदि शेष ॥ २ ॥ जाता ॥ १ ॥ गृहुर्ण ॥ २ ॥ रूप

संगुलं ॥ १ ॥ एष्ट ॥ संकलिते प्रत्ययां पदश्चीना उभये स्थापित्वा रूपोपना-
करणे फलं ॥ २१ ॥ दि फक्योपलेशपरि लिखिते ॥ योजनम यूर्तम् १७ (दूर्ते भार्त्तिमर्ति)

(ii) सूत्रम् ॥ आधोविषेष हिगुणं च य शुद्धिविभाजितं मूर्त्तिधिकं तथा
कार्य गतिसारं कताव्येतु ॥

(iii) उदाहरण ॥ द्वितीय तिमोकाल्यानं द्वितीय त्यादिकोन्तः ।

हयोच यथार्थे पत्था केन कालेन सास्याताम ॥

स्थापनं क्यते ।

| आते २ | उते १ | पुर ० | हिर ० | आते ३ | उते २ | पुर ० |
| १ | १ | १ | १ | १ | १ | १ |

करणम् ॥ आधोविषेष...
C 1.

(5 recto, 5 verso)

\[
\begin{array}{cccc}
\text{अ} & \text{०} & \text{०} & \text{६} \\
\text{१} & \text{०} & \text{१} & \text{०} \\
\text{२} & \text{०} & \text{१} & \text{१} \\
\text{३} & \text{०} & \text{१} & \text{०} \\
\text{४} & \text{०} & \text{०} & \text{१} \\
\end{array}
\]

करणम्। आधोविशेषम्। आदि "यच युद्ध यचं ६१॥ युद्ध।।| १ | आदि- \\
| शेष ॥ दिगुण।।| १० | उत्तर विशेष ३ \\

विभक्तं| १० | सहचर| १३ | अनेन का...समाध्यन मत्तित॥
प्रत्ययम्। हस्पणा-करणेन फलं| ६५ | एष पदं...अष्टादशम सूत्र १६॥

(ii) (सूत्रम)॥ दिन गमनमादि रहितं दिगुणं तन्त्वोत्तर यणा संयुक्तम्।
प्रतिनिधित्वात्यंगुणं सर्वं केष संज्ञको राशि।
अष्टोत्तर गुणिते केष संज्ञको दाता सूत्रं
प्रतिनिधित्व युतं दिगुणोत्तर भाजितं

............हृतं ३०...दिन गमनमादि रहितं दिन गमन योजन ६ पदच
| ५ | आदि। ३ | रहितं जातं | २ | दिगुणं | ४ | तन्त्वोत्तर यण संयुक्तं
| ५ | अतिग्युणं। ६५ | एष केष संज्ञको राशि अष्टोत्तर संयु... 
| लम्ब राशि | ३० | अष्टगुणं | २४० | उत्तरेण गुणमुत्तरं | ४ | ..
| गुनितं जातं | ५६० | केष संज्ञको दत्ता। तत केष संज्ञ...६४ | युतं
| जातं | १०२४ | अत्य सूत्रं | ३२ | प्रति निधित्व... | ६ | युतं जातं
| ४० | उ... ... ... ...

C 2.

(6 recto, 6 verso)

(iii) शिके प्रत्ययं| १ | ५ | ५ | फलं...अनेनसह ५५ एष समाध्यानम्॥
(ii) उदाहरण: आदि पञ्चमुत्तरं वीणि नरो योजन सम्यक्।
द्वितैयः प्रति दिनं स्पष्टः गतस्य दिनं पञ्चमुत्तरम्।
केन कालेन समातं कथ्यतां गणकोलम्।

<table>
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<th>प्रति गति ४</th>
<th>दिन ५</th>
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<td>२ ३ ४ ५ ६ ७</td>
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</table>

पञ्च दिनम् योजनीकं योजनं | ३५ | करामम्। दिनम्यनमादि रहितं
तत्र दिनम् गमनं | ४ | आदिरहितम्। आदि | ५ | रहितं।

(i) **अनेन रुपमिन्तं जातं** | ४६० | संस्करं दर्शत्रा तत्र क्षेत्र राशिः | ४६ दर्शन जातं | ४६८ | **दान ददाति सम्मू। करणो क्रियते।**

(ii) सूचना: अक्षमये शिलस्त कल्याणा शेषव्यक्ते दिस्तुंस्मृत: तद्वर्षः वल
संशिष्टः हुए। **शुद्धि कृत्यः।**
अनेन सूचने शिलस्तमूलमांकन स्वमतिम्।

(ii) लघु शृंखला: | २५ | प्रतितिलिखितं | ६ | अनेनयुं | ३६ | ४८ |
<table>
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</tbody>
</table>

***| २१३६ | हिगुगोत्तरं भाजितम्। **ततो**
<table>
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G 3.

[7 recto]
( १५ )
योजन ४२ शेषः दूसरे नियत तेन् ॥ ॥ ७७ ॥ एको विशिष्टम सूत्रम् १५ ॥

C 4.
(65 verso)

आ १ | उ १ | प ० | ६० |
६ | १ | १ | १ | १ |

करणम् ॥ अष्टोत्तरने गणिते । अष्टवन्त्रः । ४०० । उत्तर घनं धिन्नमादि । आदि धिगुण । द । बयोजितं । उत्तरम् । अतो उत्तरं पात्थिभवा एकं संधिति । १ । बं नितिध्य धनस्य । ४०९ । मूलं शिलष्ठ करणम्

| २९ | "वेणं" | ५८२ | शेषे |
| ४० | ५४२ | ४० | ४२ |

चतुर्वर्षा पृथक् स्थायिः । ४० । "शोष्यं" । ५३२ । तमूल वजितम् ॥ तमूलं

**** ८५०

C 5.
[56 verso, 56 recto]

| ५८० | ५६५ | गुणित जातं । ५३२ | चतुर्वर्षा पृथक् स्थायं वर्ग । १६०० ।
| ५४ | ५६५ | ५३२ | ५४९१२ |

एष उपरापात्य शेषं । ५४६७२० | वर्गं जातं । ६० |
| ५४९१२ |

*** | २१ | शेषं वर्गं । तस्यात्
| २० | २१ |

अछुते शिलष्ठ कुत्सुरान्त । शेषचिद्वेदि । संयुक्तम् ।
तद्वं वर्गं दल संशिष्टः । हृति-पुत्रि-हृति-शयः ॥
शेषचिद्वेदि । श्री-संयुक्तम् ॥
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<th>2</th>
<th>21</th>
</tr>
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</table>

| 16362 | 16362 | 2 |

C 6.

[64 recto]

| 405280 | 448004 | ***अग्नि कर्त्तव्य***
| 38724 | 38724 |

| 405280 | 448004 | समुद्र नाम । अभि निशा हृदेकु गुण ।
| 38724 | 38724 |

| 1954545449120 | अस्म गुरुङः 960000+
| 2954545449120 |

| अपनेदेहो दिलिमुः | 6 | श्रेष्ठ व्रजयं ग्रहिकाः
| 5 | 12 |

| अशास्वयं | 77 | तन्मूल वरिजं। तन्मूलं। दिलिमुलं संभाबं
| 92 | 24 |

| एष पदमे नमः | आर 1 | दक्ष 9 | पो 65 | रूपोश्च आर 49 | 'दलिता
| 9 | 9 | 24 | 24 |

| एक आदि संभूमिः | 48 |

C 7.

(57 verso 57 recto)

| अष्टोत्तर—ले गुणिते 40 दिहुनमादि च... सिद्धि | 49 | मूल | 6 |
| 45 | 5 | 6 |
श्यामचंद दीर्घगुणि...

श्रीमत अक्कले मिल्ला कृष्णन श्यामचंद दीर्घगुणि।
तद् वर्ग दल संविलास: हृदि-गुर्दिश्चूक्रित-श्य:।
अक्कले मिल्ला... तदवहि संगुणकल।

6 | तद्वर्गंत | 6 | 12 | 12 | 944

\[\text{सूक्ष्म} \text{पदम्}। \text{अथ} \text{प्र-}...

\[\text{सूक्ष्म} \text{एको राशि हिंदु स्थाय} \text{उपसे-}

C 8.

(45 recto, 45 verso)

(i) | 10225 | वलिता | 10225 | ...आदिवर: | 102625 | पदभाना
| 32400 | 65600 | 65600 |

\[\text{पदसूक्ष्म} \text{...} 6454040625 \text{अथो चन्द्रविशिष्ट} \text{उपरा:} 6454040625 \text{अथो}
| 32274520000 |

...लघुं 2 एक घनम्।

(ii) | अत 1 | 40 1 | पदु 5 | घनु 7000 |
| 1 | 1 | 1 |

...324। \text{अथस्वर्ग} 147453 \text{अक्कले}... 29743271536। \text{एक सर्व} \text{सुगुणित}

करणिः कृत्या भाजित जातं: | 99 67 | ...अंगीर्णणा गुणये... रसिं च चर्च्य जातं:
| 671250 |
(২২)

| ৩০৫ | ৩৩৫ | ৩৬৫ |
| ২৬৫ | ২৯৫ | ৩২৫ |
| ১৯৫ | ২২৫ | ২৫৫ |

| ৫৫ | ৮৫ | ১১৫ |
| ৪৫ | ৭৫ | ১০৫ |
| ৩৫ | ৬৫ | ৯৫ |

দরিদ্র যুয়েলে...

C9.

(46 recto.)

| ৪৫২২৬৪৬৫০০ | ৪৫২২৬৪৬৫০০ | ১২৬৬৫০২৫০০৪৪ |
| ৪৫২২৬৪৬৫০০ | ৪৫২২৬৪৬৫০০ | ১২৬৬৫০২৫০০৪৪ |
| ৪৫২২৬৪৬৫০০ | ৪৫২২৬৪৬৫০০ | ১২৬৬৫০২৫০০৪৪ |
| ২১৩২৩৭৩৪৩২২ | পাকা | ১১৩২৩৭৩৪৩২২ |
| ৪৫২২৬৪৬৫০০ | ৪৫২২৬৪৬৫০০ | ১২৬৬৫০২৫০০৪৪ |
| ২১৩২৩৭৩৪৩২২ | পাকা | ১১৩২৩৭৩৪৩২২ |
| ৪৫২২৬৪৬৫০০ | ৪৫২২৬৪৬৫০০ | ১২৬৬৫০২৫০০৪৪ |
| ২১৩২৩৭৩৪৩২২ | পাকা | ১১৩২৩৭৩৪৩২২ |

D1

[46 verso]

| মাটে ৫ | মাটে ৬ | মাটে ৩ | ডি৳ ক্যান অপর প্রস্ত: পারা স এ ও বিড়- | বিধান *স্থল হয় * বিদ্য প্যান থে | বিদ্য বিধান এ ছয়ি করণ | বিদ্যে বাড়ে ইচ্ছি প্যান ছি সাঃ*---
योल ५ जी)

\[\text{६५ | एके भागा...
2।}

\[\text{१७ | १७७}

\[\text{२ तो ० शता ७ पल २ तोल १ पल ६।}

उदा।। समा।।।।णिपेयी कुरानि ७। द चतिसस्।।।।रिलसमापाथ

dुरिताणि द्वयोधे।

\[\text{एककस्थ सार्थय:...}

\[\begin{array}{cccc}
| & १ | १ | १ | ३ | \\
| & २ | ३ | ४ | \\
\end{array}

प्रकोप-गुक्त्या-पलं...
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उदा।। वर्ष लि।।।।दादा पान्ज बन्द नूपो ददी।

\[\begin{array}{cccc}
| & १ | १ | १ | ७६४ | १ | \\
| & २ | ३ | ४ | १ | ।
\end{array}

D 3

(६९ recto, ६९ verso)

\[\begin{array}{cccc}
| & २ | २ | २ | \\
| & ५ | ६ | ७ | \\
\end{array}

वुक्षय...अजोस्वदः...हक्कम्।
उपरि मा शांततुला भवति चतवालिषय। दुना चौराशीयेः एतत तत्ततुला।

ढो चतवारिषयं वनित एते स्रीह्रिघः रावणः स्थापनस्य...

प्रत्यय धैराशिकेन | ५ भा। | २ न। | २१। | फ० त। ६४ |

***ईयस्य क्रियते।* ६

*यते राशिः ७ २ | २१। | फलं | ९ १ | १ |

कं तं १०५***

उदाहरणः लिनिद्वं, लिगुण लिगुणेन दुः

* * * * तदुपयताम॥

| १ | ३ | ६ | दृष्य १३। | प्रशेर | ३० | ६० एकरं | १३। |

***वाचा इति शान्तलिनिद्वं परव्या पवन्***

| ४ | ६ | ६ | दृष्य ५०० | *** | ४० | ६० | ५० | एकलं

D 4.

(68 recto.)

***पुरष १६॥*** देश दशम यानी पात्य जाता

शेषं २। एकरं २५॥

***बायी***
D 5.
(31 recto, 31 verso)

(1) त्रैंशिकेन ३ दिने २ दृशोऽ १६५ दिनो दृशोऽ
   १  २  १  २

तीयस्य क्रियते... ३ दिनोऽ २ दृशोऽ १६५ दिना फलं दृशोऽ
   १  २  १  २

१४० प्रयोगम दत्तम् सप्तः द्वैस्मृतिनोऽपि जाता।

३ सदृशः ७७ २५४ पाल्य शेखः २१७ द्वितीयस्य
   ११ ११ २१

दत्तम्...७७ एवस्मृतिनोऽपि

(2) पुनाम् सर्वभागम् भाँ ४ दिने। दृशोऽ १५ जीविता।
   १  ४+

द्वितीयस्य। भाँ ३ दिने। दृशोऽ
   १  २

****कारणम् चेतद संगुणोऽ ३ दृशोऽ १ ४ तत्त
   ११ १२

३ दृशोऽ १६ मुद्रण। ****ध्यात्रि हुत फलम्। अस्य शुभाकार
   १  २

भयं। २ १ ****उपरं गुणोऽ बादो ताव ५ दृशोऽ
   १३  २ २

...गुणवं २... ****निवेद्य भोजेनु... सूक्ष्मं।

४
(26)
D 6.

(67 verso, 67 recto.)

(i) चहे ति हिमण जा"ता।

निगंछः"प्रविशमाने चतुरारिक दतः गुन हिमण जाता

शून्य हुस्त गरत तस्य किमत सूत्रधन स्मात्।

(ii) दृष्टिका समान्य सुबंध ॥

दिन भक्त विशेषणः"हिमण क्रियते

चैव कालेश्वां विनिर्देशेत् वैरागिक विधानेन

"आदर्श च पात्रं सूक्ष्मे दत्तं च ततस्मात् ॥

उदाहरणम्। हिमण"""
(27)

D 7.

(28 recto,)

तं दिगुणं भारं...लबं १०।। पुन स्त्रिया...वेत्।।...सुन्तं... १४।।

dिगुणं जाता... आहुत्वभादो... गुण... भागस्य दिवधा X कि

<table>
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E 1.

(66 recto. 66 verso)

...एकाङ्क तु पत्थानं एक-द्वि-त्रि-चतुष्ठि...पश्यानमानयः

स्थापनं क्रियये

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प्रत्यय वैराशिकनेन
(53 recto, 53 verso)

9 9 9 9 9

(58 recto, 58 verso)

उदाहरण: पद्ध-विशाल लिपिपालघोक-विशाल च।

ढा शद्धाधिक संतभवर च ।

चंतुष्णिट्ट नव चद्धतम ।

उत्थानवेकश्व अष्टम ।
र ४।

(२१ रूटो, २१ वर्षो)

तिन्य तिने तथा। तिन में पञ्चम किन्ते।

एषां द ।

करणों ॥ कथाला। । २ । ३ । ४ । ५ । ६ । ७ । ८ । ९ ।

वाच्य तुलचित्य।

जीव लोकालेष्वा दीक्षा। कथा किं पञ्चम। ॥
परिवर्तन किये तारा वृः ताल का फल दी ५००॥

अस्व प्रत्यय तैरासिके।

<table>
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<th>२ दी ०</th>
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F 1.

[22 verso]

(ि)हिणुण हितिस्वभ प्रथमाःतीयोऽ । प्रथमाः चतुर्गुणं चैव चतुर्गुणं चैव

दस्त्वान् च शतमेकम् द्रयभुजग। वदस्त्व प्रथमे दत्तं कि प्रमाणं स्योः।

| ० | २ | ३ | ४ | दूसः २०० |
| १ | १ | १ | १ | १ |

शृष्टमेक युतः दृष्ट्या १। २। ३। ४।। शेष युक्त्या—
(11)

फलम्॥ २० । ४० । ६० । ८० । एवं २ । एवं……

आ । २० । उ । २० । ४ । क्षेत्रकरणे फलं २००

(ii) सूत्रम्॥ यदृच्छिक विन्यस्ते शून्ये तदा वर्गं तु कार्येत्।

F 2.

(23 recto; 23 verso)

(i) "तिलिगुणं"

प्रथमस्त् तु कि भवेत्।

| ० | तबा २ | तवा ।
| १ । १ |

यदृच्छिक विन्यस्ते शून्ये । च-च । तवा वर्गं तु कार्येत्।

| ९ । २ । २ ३ । ६ | प्रक्षेपे गुणितं १ । २ । ६ । २४ |
| १ । १ । १ । १ । १ । १ । १ |

प्रक्षेपं २२। दृष्टं विविधत् । १३२ । वर्गं जातं । ४ । ३३

दत्तम्॥ अतोन्यासः । ४ । ५ । २४ । ५६ । ……

एवं वर्गं क्रम गणितम्॥ अथ गुणि वर्गं के……

(ii) सूत्रम्॥ कालिकं शून्ये विन्यस्तं तदा चैव क्रमे गुणं।

(i) "हत्वा चतुर्णि"

प्रथमस्त् तु कि भवेत्।
(३२)

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कामिकं शून्ये पिन्यस्तं कामिकं १।। एष न्यास्तम्

tवा चैव क्रमेण गुणितं | १ | २ | ५ | १० | एष यु | ६० |

अनेन दृष्यं भाजितं | १ | ३० | जाता ५ | ए ।।

अनेन केष्वं गुण्ये | ५ | १० | ४५ | २४ | ।। युतं करं गुणितं ।।

(ii) उदाहिः ॥ प्रथमस्य नातं चैव धनम्।

स च द्वार्यं युतो द्वतः...

F ३.

[२४ recto, २४ verso]

शत्रुधुतुतचतवारिश्च दत्तं चैव चतुरुपगमयि X

किं प्रथमस्य...

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शून्येणु १...युतं चैव गुणं तत् युतम्य चैव गुणं कल्वं कारणे गण ॥ १। ॥

गुणम्। उपरे उपरमधे अर्थ गुणये | १० | सार्थ दू...युतं... ॥ २। ॥

स्तीय रायामुगम्। सार्थ स्पर्शिन्न तीशि | ४५ | सार्थ लय युतं...चतुर्यं ॥ २। ॥

राशि गुणपेतेषु षडं विशालिथम् जाता | २०५ | सार्थे चत्वारिः यु... ॥ २। ॥
(31)

284 | एवं दृष्टं। सचि तदेव जातम् —

(ii) ""विसार्थं यु्"

चतुर्गुणं चतुर्गृहं नवायं युज्ञम् दत्तम्।

...दिशाता द्राविषाधिका फ्राम प्रथमस्य दत्तसित्

| 0 3 | 2 5 | 3 7 | 4 6 | एकरं दत्तम् २२२ |
| 1 2 | 1 2 | 1 2 | 1 2 |

शुष्य...दत्ता। १। गुणित गुण रुपमेण जातम्।

स्थाय... | 5 | 15 | 67 | 357 | दृष्य २२२ | प्रक्षेपण
| 2 | 2 | 2 | 2 |

जातम् तृतीय २२२। ""दृष्याः..."" २२२।

(ii) उदा।। प्रथमं न जानामि। दिवार्थं युज्ञः..."

F 4.

(25 recto, 25 verso)

| 0 3 | 2 5 | 3 7 | 4 6 | दृष्य ६८ |
| 1 2 | 1 2 | 1 2 | 1 2 |

...युज्ञं जातं ५। द्वितीय गुणं ९।। तृतीयां एक्ले

| 2 |

गुणितम्। युज्ञेन। १०। २३।...गुज्ञं। ३३। गुणितं

| 2 | 2 | 2 |

| २३२। रुपमं जातं। पार्यं...एष्य न्यास। ५। ५। २३। १२३। दृष्य ७६ |
| 2 | 2 | 2 | 1 |
(२४)

| १५६ | विषतत्वम् | २ | ७८ | १५६ | १ |

(१) करणम्। गुण्य स्थाने...रूपं दत्ता १ | युता जाता | ५ | १

... १५। प्रथम तुल्यक्षय विगुणं युतं जातं

चतुर्गुणं नवार्थं युतं जातं | २८ | एकत्र स्थान... | ५ | १५ |

| २२ | २५ | २४ | ७१ | प्रक्षिप्तं | ७१ | भक्तं ह्यं जातं |

...अनेन सर्वं गुणितं तदेव | ५ | १५ | २२ | २८ | एकत्रम्

एषापरि विद्धः।

(२) उदाह । प्रथम धर्मं दत्तं न जातं कि तु दिवर्तं युग्मम्।

तदद्वित्तीयेन विगुणं दत्तं प्रथमार्धंहीनम्।

तदा द्वित्तीयेन विगुणं दत्तं सप्तार्धं...\

चतुर्गुणं नवार्थं हीनम्

दत्तेकर्तं त...***

| २ | ५ | ३५० | ७ | ४ | ५ | ६ | ३० | २५ |

| १ | २४ | १ | २४ | १ | २४ | १ | २४ | २ | २ |

F ५।

(२६ recto)

(१) करणम्। गुण्य...रूपं दत्ता: युतं जातं | ५ |
(15)

...(1) प्रथम त्रृतीयं त्रिनुपं द्रष्टमा

चतुर्थ चतुर्भुतं नवार्थं रहितं। शेषं/11ए...

...अवलं 26 जात 16 " गुणितं तदेव।

एवं अहं राशी भवति।

(2) निश्चयं "मातम्। शृणुस्थाने रूपं दत्तव। तदनुपूक। गुणित।"

F 6.

(26 verso)

अथ ची 4. 30 अथ दलं 60
अवाच 8. 32 दलं 60 60
भू 24. 16 24 दलं 60
24 16 दलं 60

24 4 अथ सौन्तुऱ्यां द***

26 4 36.20। 4 अस्य लिव***

32 4 32.20। 4 अ***

36 4 28.20। 12 पुनी***

भू 26 24.28। 16***
(10 verso, 10 recto)

(i) सूर्य २४

(ii) सूर्य ॥ कृत्वा रूप क्षण पार्थ धान्त संगुणानं ततः

प्रत्युत्तरणं ततः...विनिदिशत् ॥

(iii) उदाहरण ॥ विभाग मलदश्य...विधानस्थैर्... अष्टोत्तर-ञातानि दर्तं कि केषष्ण वद पण्डित ॥

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कृत्वा रूप क्षण पार्थ जाता | ३२ | केषष्ण ॥ प्रथम व्याख्ये

क्षण | ३६ | केषष्ण | ७२ | द्वितीय व्याख्ये क्षण | २४ | केषष्ण ४३ तृतीय- व्याख्ये क्षण | १६ | केषष्ण | ३२ | प्रत्ययं क्रियते। स्थापनम्

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समाप्ति क्रिया

(i) ...०... जिब्बि वतन्त्र-भाग संयुक्त...}

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(ii) उदाहरण। सहस्र धातुस्य लोहस्य दशाङ्का कृतिते-स्वर्णाम्।

सप्तते द्विगुण च कि शेषं वद पाण्डिता।” १ ६ २। १४० १।

कृत्वा रूपस्वरूप प्रस्थापिते। रूप १ ३। क्षय कृत्वा।

जातं शेषं ० ७। पूर्ण ३ ३। अनेन गुणितं जातं ४ ०। क्षयं ४ २।


eवं १४०।

. . . . . . ० ० १ ३। पूर्ण २। फलं १४०।

G 2.

(१1 recto, १।1 verso)

(i) प्रत्येक: ० १ १ २ ३। २ १। ५ ०।

(ii) उदाहरण। . . . . पला कृते पल द्विघात्य क्षय भजति।

अष्टावस्था . . . . यतं बृहि।

| १ ५ २। १५। ।

करणम्। अवध्यस्य पल्लवेदेश्य इत्यं २। कृत्वा रूप।
(३८)

(इ) क्षय १ । क्षय कृत्वा जात । ६ । १ । गुणेत्त जात । १५ ।
क्षय । ४ ।
प्रस्था तैराकिते ॥

बृहत्यवर्ग पल क्रीते विभाग क्षय गच्छति ॥

अष्टादश पल क्रीता किं क्षय वद प्रणित ॥

| १ १ | १५ । पल ४ । |
| १ २ | १ । |

पुन विभाग दिवर्ध तदा चतुर्भि किमिति

| १ १ | ४ । पल १६ । |
| २ २ | १ । |

(iii) उदाहरण ॥ चतुर्भिग मल दशक सुवर्ण शालप्रचक्रम ॥

स्मार्क प्रत्यय ॥

| ० | १५० सूर । पल मूल १५० ॥ पुनर्भ्य प्रस्तार क्रमध् |
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| ३ + | ६४ । |
| १ + | ६४ । |
| १ + | ६४ । |
| ४ + | ६४ । |
| ४ + | ६४ । |

वर्ग चतुर्भि प्रत्यय किरदर्षे

| ० १ । १ । १ । १ । मा । शेष १५० । पल ५०० ॥ |
| १ १ । १ । १ । १ । ४ + ४ + ४ + ४ । |

शेष १५० तो १ शेष १ ।

| ३ + ४ + ४ + ४ । |
| ४ + ४ + ४ + ४ । |

अन्य संख्याएँ
(१५)

ग्रंथ काळ १२२६ शेष ३७५ । दूसर्ये काळ ६२ तौ ३ मास ६

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शेष १५५ संदर्भ कलंकं ॥

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G ३।
(१२ recto, १२ verso)

***प्रस्थ ''''मधुनास्तथया:

अन्याय """

कीत्व रूप काळ पार्थिमिति तत्र काळः पार्थः: हृतः: तत्र कालः

रूप गुणय शेषें | ३ | ३ | ३ | ३ | ४ | ४ | ४ |
|-----|-----|-----|-----|-----|-----|-----|-----|
| गृहूति गृहूति गत्वानास्य ।

पिवेद् '''' गृहूति योजनम् । चतुः प्रस्थै आहकम्। तदा यान्त्रसोऽस्य....

तत्: | ६९ | आवृति प्रवृतिमूँणम् तत्: | ४ | अनेन गुणितं जालं | ६२ |
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एष मध्य भागाभागे हृते लक्ष्मण । मधु प्रस्थ १ कु० १ के | १ | अन्याय |
| १६५ |

भाग प्रस्थ २ कुडवः २ के | १५ | एवं | ४ | """"कुडवत्ति प्रलोकः आहका |
| १६ |

वोभल कुडवा प्रवति | १६ | अतो मस्ये शेषम् १२

(१) ''''''''प्रस्थ कुडवा | ४ | ४ | शेष चतवार् ''''

कुडवः | २ | २ | ४ | ८ | पुन
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(40)

| (ii) उदाो ॥ वटव शुलक चतुर्मृणमण्ड्य अनिैत कुष्ठ मा । ।
| चु शुलक शालेस्तु किं शेषं वद-प्रिति ॥ ।

| १ | १ | २ | २ । ।

करणम्। कुला रूप-क्षरं पास्तं पास्तं | २ । ।

जातं | ६ | शुलके | २ | शेषं | ६ | १ | अनेन गुणितं जातं | १ । ।

| ४ | १ | २ | २ । ।

(१) उदाो ॥ अजातारभ-लोहुस्य दिन-चुः-पख्वक क्षये ।

सप्ताचकलित पिण्डस्य लिखाण्तकोष्य दृष्टये ।

किं सर्व वद तत्वं क्षरं च मध्य कपियताम् ॥ ॥

| १ | १ | १ | ३ | ४ | ५ | ६ | ७ | । ।

करणम्। कुला रूप क्षरं पास्य | २ | ३ | ४ | ।

जातं | २ | रूपक्षरं | ३ | अनेन शेषं भक्तं शेषं | २७ | ।

भक्तं जातं ४५ अस्य सप्तालिव । पात्य शेषं १५। । पूर्भ क्षयम् ॥ ॥
(iii) उदाहरणों का उपयोग साधारण चिह्नों के लिए पद्य रूप से लिखिता है।

न जानिए तुम प्रजातां न तु शोष प्रदूषितोः

प्रवृत्ति शोषण थो पिण्ड केवल विषाणु विषयम्।

अजजात्कां प्रवृत्तीस्यां कै च शोषण वद्यमे।

| 1 | 1 | 1 | कूटवा |

G 4.

(13 recto, 13 verso.)

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| 8 | 1 | 3 | 3 | 3 | फलः | 51 | पुनाव्यः | 32 |
|---|---|---|---|---|-----|-----|---------|
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ग ५।
[14 recto, 14 verso]

( ४२ )

पुनाय्य प्रत्ययं

फलं ३६ II ***मूलं न जायते

फलम्

वत्स तन्मययता चक्षु

करणम् II क्लो ल्ल्पा क्षयं पास्त || २ || ३ || ४ || ५ || जातु संगुख्य

जातं || २ || एताद्विधि रूप संशुद्धा जातं || ३ || अनेन भक्तच गुलक

पििण्ड गुणितं जातं || ४० || एष पििण्डम्

प्रत्ययं || २ || ४० || गुणित जातं १६ शेषं || २४ || एवं || ४० ||

अन्यमस्त्य प्रत्ययं || ४० || फलं १६ क्षयं १६ एवं ४० ||
(II) उदाहरणः।। गुड़पित्त जात तुल्योपचतुः अथवे गुड़म् ।

तिच्छु भ पतन्त्र पद्म वृद्ध्या चत्वारिश (भः) वे क्षय

G 6.

[15 verso, 15 recto]

"प्रबृति भवेत् सके"

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करणम्।। धान्तो धात्तित तेन। रूप क्षयं कर्तवा जात

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<td>गुणित जाता</td>
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विशिष्ट त का संचण:

चतुप्रान्त लोहस्थाग्राहीतिष्क दत्तवान्।।

कि शेषं वद धर्मं य गणिते कृतं श्रमम्।।

| २१ | १ | १ | १ | फलं शे० १६ ||
|---|---|---|---|---|
| २ | १ | १ | १ | +
| २ + ३ + २ + ३ |

पुन प्रत्ययं क्रिययं मूलं न जायले

| ० | १ | १ | १ | भाप० शे० १६ | फलं लोहपल २१ ||
|---|---|---|---|---|---|
| १ | १ + १ + १ + १ + १ | १ |
| २ | ३ | ३ | ३ | ३ |
उदा॰

कश्चिं यदि शक्य तद्वच्चयताम् ॥

एतर्मेव संशयं प्राणदू धार्मिक्यं विचारणा:

| २ | १ | ४ | ॥ ॥ ॥ ॥ ॥
| ३ | ४ | ५ | ॥

करणम् ॥ धातृ संगुणं गुणितं जातं ॥ ॥ रूपं द्वरा ॥ ॥

भागे हुते लद्धम् भः ॥ ॥ ॥ ॥ ॥

वेयं १२ ॥ ॥ पञ्चविश्वातिं तृत्यम् ॥ ॥ ॥ ॥

G ७।

[16 recto]

(ि) विभक्तं जातं ॥ ॥ ॥ ॥

भागे हुते लद्धम् ॥

भरत्व प्रश्यं तैराशिकेन

| २ | १ | १० | ॥ ॥ ॥
| १ | ॥

(िि) उदा॰ ॥ मार्क्कक गृ-चटकस्त्वेय द्विति-भाग प्रवचितम्

द्वितीये द्विपञ्चमो-भागो तृतीये द्विपञ्चकोद-भवम्

चतुर्थे द्वि-निवं भागस्वं जातं पल लयम्।

वणुवासौिलकः हुत्वा कि सर्ध्य यद परिषिद्ध॥
(४६)

H 1.

(16 verso)

सूर्यम्

(िना) इवानि सुवर्णस्वयं वच्यमित्वः स्वेदस्त्र

(िं) सूर्यम् [[वेष्क्षणं संकाश्व ननि दुत्तितितिचिन्तिति ततः]]

संयुतेतरं कनकैरककर्ष्य कशो हि सः ।

(िंं) उदात् || एकः-द्रि-त्रि-चतुसंख्या सुवर्णा मापकी चतुर्थी ।

एकः-द्रि-त्रि-चतुसंख्या रहिता सम्भायताम् ||

स्थापनं क्षयते । एवं । १ + २ + ३ + ४ + । ९ २ ३ ४ ।

करणम् || "कशो संयुत्तम कनकार्णिमित्" श्चेति संयुत्तम जातः । १। ॥ ५ २ ९६ ॥

एवं गौति । ३० । कनका गौति १० अनेन भवति लक्ष्मणम् ।

H 2.

(17 recto., 17 verso)

(िना) १ । १ । १ । १ । २ । २ । ३ । ३ । ४ । ४ । ५ । ५ । ६ । ६ । ७ । ७ । ८ । ८ । ९ । ९ । १० । १० । १० ।

(िं) उदात् || एकः-द्रि-त्रि-चतुर्थी संख्या सुवर्णः प्रोज्ञितां इवे

मापका द्रि त्रि-चतुर्थी चतुर्थी परे वरकराश्चकम् कि क्षयम्
करणम् ॥ "क्षण संगुण्य कनका" एष स्थापयते।

करणम् ॥

"तद्युगितांभाजीत् ततः" हि सास्ये शुद्धे गुणम्। ९६३। "संपूर्ति X कन्दा्"

भक्त्वा तदा कनक। १०। अनेन भक्ति जाता। ९६३। एष एकै नुष्णम्।

६००। क्षयम्।

प्रत्येक वैराजिकेन्।

| १० | ९६३ | १ | फ० | ९६३ |
| ९ | ६० | १ | ६०० |
| १० | ९६३ | २ | फ० | ९६३ |
| ९ | ६० | १ | ३०० |
| १० | ९६३ | ३ | फ० | ९६३ |
| ९ | ६० | १ | २०० |
| १० | ९६३ | ४ | फ० | ९६३ |
| ९ | ६० | १ | ९५० |

करणम् ॥ यथा माधवि उत्तरे एक हृदनारम्।

सुखम् ॥ यो सम्भव्य कथ्यता गणकोत्तम।

स्थापनम्।

| ५ | ६ | ७ | ८ | ९ | २ | ३ | ४ |

"क्षण संगुण्" जाता। २०। ३०। ४२। ५६। ७२। ८०। २। ६। १२।

एवं युति। १३०। कनकानं युति ४५। अनेन भक्त्वा लब्ध। १६०।

पुनःद्वियमाहां च नप्रत्येके। फलं। ६ फ०। १। एष एकै माशक क्षयम्।

प्रत्येक वैराजिकेन्।

| ४५ | ३३० | १ | फलं | २२ |
| १ | १९ | ३ |

एवं सर्वोऽपि प्रत्येकम्।
H 3.

(18 recto, 18 verso)

(i) (सूत्रम्) II अप्राप्त संगुण रतिि कल्पनानिन्तीलोकिणिः

... ... ...

कल्पनानिन्ती यददे भवेलंक्ष्य स क्षय भाग मापक II

(ii) चदार II एक-दी मापको प्राप्तो द्रोष्ट्रो च प्राप्तं च पञ्चसंभिः

तयास्म कवितिः फ प्राप्तं पञ्चसंभिः न केलम्।

चतुर्भिमिखलिनीं कवित्र दुष्टव्र मया संहे।

तयास्म कवितिः फ प्राप्ता सुवर्णां मापको बदः।

\[
\begin{array}{cccc}
1 & 2 & 3 & 6 \\
2 & 5 & 0 & 4 \\
\end{array}
\]

कर्णम् II "अप्राप्त संगुण मातीदिदम्" ति || 6 || अप्राप्त कविता चार्जार। 4 |

संगुण जातं । २४। "कल्पनानिन्ती लोकिणिः" त्रायस्मिक पञ्चसंभिः द्वयं संगुणय जातं

\[
\begin{array}{cccc}
2 & 10 & 12 & 12 \\
1 & 4 & 0 & 12 \\
\end{array}
\]

हितम् २ "हितम् जातं शेष १२।। अप्राप्त गणिबी।"।।

....... अष्टविशालिनस्मृतिः

(i) सूत्रम् II अनैसरसगुण ननका ततु पिण्डव्य विशोधयेतु।

चु नवं कनकाम्यास्ता शायशं शेषम् विभाजयेत्।

अप्राप्त गणिबका शेष शुद्धेन कनकेन तु।

यस्तं ततु प्रमाणं तु गणिबका या विनिदिशरस्।।
(iii) उद्वा० ॥ एक-दि-दि-चवुससंवर्य-अर्थां नाशाकानि तु
एक-दि-दि-चवुससंवर्य एकत्रायित्वा किलः।

गणिका नात कनका अनकादश माषिक।
अर्थां नात कनके प्रख्याियर्**यः।

\[ \begin{array}{cccc}
1 & 2 & 3 & 4 \\
1 & 2 & 3 & 4
\end{array} \]

करणम्**रूपस्त।।

J 1.

(30 recto; 30 verso)

... ... ...

सुल्क। एक मुह नरः सर्वर् वह्य प**रयर्

***अनेक लब्धयः हीताप्रमृः

\[ \begin{array}{cccc}
36 & 42 & 48 & 54 & 6 \\
36 & 42 & 48 & 54 & 6
\end{array} \]

...स्वर्ण क्रियासूचना हर्ष क्रियान्ते | 234 | ... |

...तुलाड़े | 3 | मुद्रा गाढ़े 1 | क्रियाते |

\[ \begin{array}{cccc}
30 & 24 & 43 & 70
\end{array} \]

J 2.

[65 recto,

एतत् = काल** चिर्मुखा य***लघुस्तः**

अपर प्रश्नः
( ५० )

यथाक पुश्पश्य द्रम्मापु-पट्ट विशेषितिने जीवनोका। तत्कार्य प्रस्तुत।...

स्पैशिवानो... नापाकरासकाना द्रम्मापु-पट्टिके कात्य दिना जीव तोऽक प्रभुति...

करण्मु। आदि तावथाक पुश्पश्य द्रम्मापु-पट्टि-वितं शास्त्री... जीव्यः

तत्तु सप्तसतीना किम्

| १ स्वदो ५ | ३० दिवो ५० पुषो | फलम्***

**भम्मा***नरीण धार्मिकता******

J ३।

(41 recto; 41 verso)

**भम्मा अष्ट द्रा-चत्वारिशधिनि। तत्तु सपति***

य ४२ दिने | द्रो १ जीव्या | ६० पुष्या ५५***

भम्मा ५६० ॥ यदि पच्च-सत्त-पद्धारधिक***द्र-चत्वारिशधि तत्रू भम्मे,

अष्टधि कात्य दिनां***

... २ अष्टेतपये दतः | १७ | अष्टेनो परि सं***उपरिम

| २ | ३ |

राशिः द्रु गुणये | ५१ | ......... उपरि युक्त क्रियते एक पत्रवाराना ५१

| ६ | २ | ३ |

स्थापत्यः १ | ५३ | फलं भा १७ वि***२
(५७)

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(51 recto.)

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(ii) उदाहरण द्वारा भारतीय पद्म नि-दिने नव भक्षये

भाद्रागार तथाय लिखि कि काल सार्वं भक्षणम्

<table>
<thead>
<tr>
<th>दी १</th>
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कश्यप् "भाय-भय विशेष्य भु"। तत्रायं ५ ॥

(i) "बोधि। फलं ९५; हृदय-प्राप्तम् तुलयम् ५२ ॥

(ii) सूत्रम्। अह द्रष्ट किं विशेषं विभाजयेत्

यल्लवेष्य हिन्दिनं कालं दत्ता सम-भन्ना प्रति ॥

(iii) उदाहरणकाल समधना भावनित ॥

द्वितीयं पद्म दिवसे दस मार्गये बुधः

प्रथमेण द्वितीयस्य सप्त दत्ता निधानात्।

दरवा सम-भन्ना जाता केन कालन कथ्यताम्

<table>
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L 2.

(६१ recto, ६१ verso)

(i) "अग्रेन कालन समधना भंवति ॥

प्रथमेण जै-राजिनेन क्रियते

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</thead>
</table>

४३। यूँ समधना ब्राह्मण।
(५१)

(२) उदाहरण: राजपुत्री द्वारा केवल नृपतिस्तव की वृत्ति के:

मेकास्यालाले द्रव्यश्रध्य यज्ञा भागा द्वितीयस्य विवर्धकामृ।

प्रथमेन द्वितीयस्य रश्व दीनार दत्तवानृ।

केन कालेन समतामृ गणयित्वा वदानृ मे।

<table>
<thead>
<tr>
<th>९</th>
<th>५</th>
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<tbody>
<tr>
<td>१</td>
<td>२</td>
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</tbody>
</table>

करणम्। "अन्ध द्रष्य विशेषपञ्च"। ततव

(१)

| १ | १३ | ३० | फ ६५ | प्रथमेन द्वितीयस्य
<table>
<thead>
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<td>फ ६५</td>
<td></td>
</tr>
</tbody>
</table>
| १ | २ | ३० | फ ४५ | ...

५५। ५५। समाधाना जाता।

(२) सूत्र विन-पन्नाशाम: सूत्रम् ५३। २०। ५

सूत्रम्। विकृतियान क्राप क्षेत्र भाष्यं रूपं हीं गुप्तमेनेत्र

लाभेन गुणये तत्त नीवी भवति तत्त चै।

(३) उदाहरण: उर्मि X क्रीणाति वस्त्रस्तु विक्रीणाति विभिन्न पद्द

अष्टाब्के भवेदल लाभा का नीवी तत्त क्रमेतानम्।

| ७ | ६ | १५ | लाभा
<table>
<thead>
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<tbody>
<tr>
<td>२</td>
<td>२</td>
<td>१</td>
<td></td>
</tr>
</tbody>
</table>

करणम्। वि.......

L 3.

(६२ रेक्टो, ६२ वर्सो)

नीवी जाता। स्य प्रस्थय बैराणिकेन।
यदि द्विभिस्सप्त लघ्यते। तद्व चतुर्दशितिभि ४४ \text{किम्}।

\begin{align*}
| २ | \, ७ \, | \, २४ \, | \text{फल्य ४४} & \\
| १ | \, १ \, | \, १ | \end{align*}

(१) अस्व विक्रयं कियते। यदि प्रभुं तथा लघ्यते तदः चतुर्दशितिभि \text{X किम्}।

\begin{align*}
| ६ | \, ३ \, | \, ४२ \, | \text{फल्य ४२} & \\
| १ \, १ \, | \, १ | \end{align*}

मूलं ४। पात्य्योष्ण १८ \text{एष लाभः}

चौपचाचार वृत्तम् ५४

(२) सूत्रम्। विक्रयं माज्ये चैव गुणयेत् क्रय पिपलाम्।

रूपो नेम मूल गुणये लघ्य लाभम च प्राप्यते॥

(३) उदाहरण। द्विभि क्रोणानि यस्मि विक्रोणानि विभिन्न यदि...

(१) \ldots २ | \, ७ \, | \, २४ \, | \text{फल्य ४४} | \text{अथ विक्रयं} \, | \, ६ \, | \, ६ \, | \, ४२ \, | \, ४२ \, | \, २४ \, | \text{पात्य्योष्ण १८} \, \text{एष लाभम्} \, \\

फलव पपचाचार वृत्तम् ५५

(२) सूत्रम्। विक्रयं माज्ये चैव गुणयेत् क्रय पिपलाम्।

विभिन्न स सत्तैव गुणये निधिकं बुधः।

वल्लभं सा भवेन्मूलं यज्ञेषु लाभपिपलाम्।

(३) उदाहरण। \text{द्विमिश्र लघुरूपेष्टी चतुर्भिश्र विक्रयः षट}

स मूल लाभमुलमा शतं पाठि \text{द्विमिश्रितम्}

किम मूलं क्रान्त लाभम च कथयेदु गणकोणम्:॥

\begin{align*}
| ६ \, | \, ६ \, | \, \text{सिध्य १६} \, | \\
| ३ \, | \, ४ \, | \end{align*}

करणम्। विक्रयं माज्ये चैव गुणयेत्।

\ldots \ldots \ldots \ldots \ldots \ldots
(63 recto, 63 verso)

(i) ⋯ ५ १ २४० फल ६० ⋯

| ६ | ४ | २४० | कल १६० | मूल ५० | पाल्य शेष ७० |
| १ | १ | १ |

(ii) षट्पञ्चाशाम सूतम् ५६

"विनम्रयुक्त विहंसक्युं गुणित क्रम राशिवतु।
कृत्वा क्रम कर्णण्वृत्र विभक्तं सूतं आप्नुरातु।"

(iii) उद्यो। पञ्चमिश्रतु वर्ग तु गृहीतं 'केन मानव।
केन गु ष्टि विकीर्तं गु ष्टि पञ्चाशा ऋणं क्रतमु।
क्रम विनाय संगुण्य नीवित्वस्यैव कथ्यताम्।

| १६ | ६ | क्रतम् ५६ । |
| ५ | १ |

⋯भाजये चैव ।

(i) पुनास्य विक्रय । ६ १ २५४ फल ६४ मूल १२० |

चुतुष्णि तात्य शेषम् । ५६। एष ऋणं क्री⋯

सप्तपञ्चाशाम सूतम् ५७ ॥

(ii) सूतम्। वस्त्र शुल्कं यद् भवति तदु⋯हृत कस्त्रताम्।

दैराधिक विन्यासेन शुल्क विक्रय तर्कत्। ॥
(३६)

(iii) उदात् ॥ पद्म शुल्क विषयां प्रकरणम् ॥

पदकाणां यान कृते द्रो पटी हुत शौलिक।

...सुल्तं पण द्रश्यतः तेषाम् कि सुल्तं ...२० recto, २० verso)

M ।

(१) १ २० रक्ति धार। १ | सु० १ छो। द०! रक्ति-मू० रक्ति १ | फ०म० ४
| २ १ | २ १ | १ | २ १ |
| २ १ | २ १ | १ | २ १ |
| १ १ | २ १ | १ | २ १ |

पुनः तदायुष्य २ २० | १ | १ | २ १ | २ १ | १ | २ १ |
| १ १ | २ १ | १ | २ १ |

...छेंद्र ६ धार।—द० फ० धार। ४ य० १ ... पा० २ सु० १ ॥

सुर्यवध्य मान समा...।

(१) उदात् ॥ स पनचनव प्रायानिनि दिनानि तयोदश।

... नाम् किम् ॥

(१) मू० १२०००...।

उदाहरणम्। सर्पोद्त-दश हुस्तो प्राविष्कायार्थिगुलम्

स नव भावं ति एक विशालि प्रायमपहारानि ।

प्रतिदिनेनः कि किलेत चिलं संप्राप्यते ॥
( 33 verso, 33 recto )

उदाहरणम्। कीट X: किलाघुलं दिवसे दिवसे***
...
...

M 2.

( 33 verso, 33 recto )

उदाहरणम्। कीट X: किलाघुलं दिवसे दिवसे***
...
...

(ii) उदाहरणम्। दीनार कोनागविषा तिरु X खाजांनीयं सुखवाचने च।
तस्यायांमध्येत यद्यमध्ये तत्त: देव गुहु प्रसां ऐपणधन भुक्तम्।
...
...

(ii) उदाहरणम्। अर्थ ऐस्तार नेप्र रोमोशतानि च।

ढासस्त्रीति-चर्मणि कटि-रोमाः...
(i) चन्द्र निघाण

पुस्तक में नीति रावणे। पुस्तक में त्यसू दुःखशेषत्या। साकृंक रावणर्थ
दर्शनक श्रद्धवर्षेवने पतमाणसी दश भागं निधार्यते। एवं तत्त परिमाण हीय मानन
तु निष्ठाम्: कियतस्तुरावर्तेन भूमि प्राप्यते ज्...

<table>
<thead>
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<th>1 1</th>
<th>पररा 5</th>
<th>योजा 30</th>
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<td>5</td>
</tr>
</tbody>
</table>

फलं पररा 29-157 शे 5 11

(ii) उदार ॥ नागश्वच्छन्द गामि प्रति च दश

(i) 100 उद्धेष्टेऽ 768000 वहो-यो 1 अधण्डेऽ 768000 फलम्
| 6 | 1 |
| 3 | वहो 423-67 मात्र 1 दिश 4 |
| 40000 |
| 5 |
| 29 |

(ii) उदार ॥ अवज्ञं चरिश्वाय्का भाृि भूमि तले पठम्।

निष्ठादेशां ॥ नाय्त्रु सस्त योजन हीयते।

चतुर्दशस्तु कोटि हृयत पहन-पष्टि च।

कै दिले भूतले प्राप्य वदे गणकोठम॥

भ्यास स्थायन प्रयति।
( ५४ )

M ४.

( ३६ recto, ३६ verso )

(i) छा पञ्च बिद्युतित सते

...... एष देश प्रमाणं समाप्तम्।।

(ii) उदाहरणस्य राष्ट्रे कौट्यां वा क्रजां हैन।

एषं चैकं राष्ट्र पुनस्…… पद्धा नीता।

सप्तांत्रो विपि चैकं राष्ट्रस्तुतिवर्गयति।

पञ्च सप्तांत्रया ……। भवेत् सप्तांत्र गुणं किम्

| रा १ | १०३५ | ५६ | अनुवर्तेण २०००* प० माण । फा मा १० ।
| प० २०० |

एष राष्ट्र लब्ध्यं प्रमाणम्

(iii) काकिनी दश मण्यस्य द्वादश्य वाष्पीति……

तस्यं विशिष्टं भागस्य शतं भागं प्रवचन्ति।

नरो वशस्य……

(i) | यो २ १ १ + | यो ५ छो ४६०००० + यो २० यो ४ १ ३६० ।
| २१३३३ माण ४

(ii) योजनस्य तिभागाध्वं स तिभागपदेनकम्।

या नौ विनिमयः ग्येन गच्छति।

सा पुनः पञ्च भागाध्वं योजनस्य तथापटमम्

……ति निकर्तुः वामुवेगवलाहता।
(६०)

योजना नामितो तर यत्त के सेन कालेन गच्छति ॥

| दि १ मा १ १ ९ | वा १ १ १ १ १ १ १ १ । ।
| २ २ २ २ २ २ + २ | ।

M 5.

(34 recto, 34 verso)

[१] ख गा एकाष्टश सुंदर प्रशुंगति पौनःमेव च ॥

. धौटी वद वशे कि खगस वद सुन्दरि ॥

| प्र १ | १ | ख ११ | खा ५७६० | विल खगः ६३३६ । ।
| १ | १ | १ | १ | ।

एष बाद्र प्रमणामु ॥

(२) कवित्त पुराण सुवर्णस्तु कला पाद युतं यवम् ॥

प्रश्रुत सुविले सुबिर फिल दर्शावनु ।

पपाध्याय भासमेवस्तु दिने पप्पदशस्त्रायः:

dर्थाः सर्वव यातुमिच्चायीमि तत्सच ॥

| दि १ । १ । ६* वा । ६* । १ । १ । १ । १ । ।
| १ । ४ । ४* । ।

(१) ... ...

चित्तिताः । तानीयत्तार-परम्परायज्ञिन दुर्ग ॥

स्यंशास्त्र । १ | खा १ | योजः ७७७ । १ । । विल — ६४० । ।
| १ | २२२ । २३२ । ।
Specimen of the Bakhshali Script
M 5. (34 verso)

M 5. (34 recto)


(61)

(i) माशाहार्य युतो हृद्यः—विस्त पश्चापन्नचाण सतिरेण वधा नमः

ि० १ च० ६० १२८* मा० कृ० १ मा० च० ४०* सि०-मा० | स० ५२ |

२

M 6.

(37 recto, 37 verso)

(i) …सूत्रमानस्य

विवाकरस्य घटिकाः X दश प्रयातः वद विशिष्टम्

| ३० म० | ८० | ५००,०००,००० | च० १ | फ० यो० ३७३३३३३३३ | १००००००० | २ | ५००००००० | २ | फ० यो० १६५६६६६६६६४ | २ | १००००००० | २ | ५००००००० | २ | १००००००० | २ |

(ii) भानो रघुं सुर महोरण सिद्धसंग्र विदाधिः फ परिवृत्तम्

अहोरात्रः। कोटी गारां स रघुं प्रत्यथातः तदु बृहि शास्त्र–

कुशालो……तत्तम्।। गुर्देश्वे केवल फ गच्छे बृहि ने मणकोर्तमा।।

(i) "भा० प्रथेद् राशी ।

उर्ध्वच्छेदं १०००० विलिप्तानां……लिप्ता ५

(ii) पश्चापे संवत्सरे भुके गारैका यदि भावनजः बृहि……क तत्तब

समस्त वास्तरे किम्

| २ | रा० १ | १ बं० १ | २ | रा० १ | १ बं० १ | २ | रा० १ | १ बं० १ | २ | रा० १ | १ बं० १ | २ | रा० १ | १ बं० १ |


(६२)

अध्यादेव १०८००० विलिपां राणिः अध्यादेव १ विलिपां लिप्तं ॥

फलं विलिपां २ ॥ एष प्रधा गतिम् ॥

(१०) उदार ॥ राज युधिष्ठिरो नाम: पाण्डवंश ॥

M ७।

(47 verso, 47 recto)

(१) ... युधुः पार्थेहृतको प्रत

सायकेश्वरं परं पद्मं स्वपाद दल घोड़णि ॥

अत्यन्त चतुर्या वै हुना तेन महात्मावान ॥

शरणां च परिमाणं...विषारद ॥

| विष १ | १६ | ४ अं ५० | २१७७० | फलं घर २६२४४०० |
| १ | १ | १ | १ | २ |

अन्यं ई प्रमाणम्

(२) सूत्रम् ॥ एको रथो गज्

(१) ... ... ... ...

... ... विश्वामणः

चम्पस्तु पूर्तनाःस्तिप्तस्तिलश्चम्...

अतीकीनि दश्यां आहु । 'अहर्क्षोहनी बुध: ॥

अश्वोहि...
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(ii) उदाहरण । कविचदू राजकुमार शादुदम ।

M 8.

(48 recto, 48 verso)

छेदमू ४०* रत्कु-पल……गुणित जातं ४१५८४२……3६ पल

191200

tैर० ५   पले २००० तोल-पल १२ तैर० १२* धारा ७ धाने नासित* ४०*४०*२

(i) । पल भार २……एनासित

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रक्ति क्षय...पञ्च गुणम्

दिवसा विशेषता कि गुणित महां... वद निश्चयम्

62329 | क्षय शोष्य... | 600=91 | अधज्जेदं 2000
1620 | 56200 |

सर्वणुितं 15=275670 | "7 तोले पल्स..."
156200 |...
(i) य० ३ यववास्तिको ६
 १ द० कलास्तिपा ५
 पादनास्तित मू० ४ पास० ५० २

(ii) उदाहरणम्

...शुश्रूषे जयिति देवी प्रतिमद्विशेषेन ददामि देव्या...कथम् कृत्वा दीनारः
शतास्तिवारित धानकाणिका रत्नकायवाकलापादपुरुणका च। एतदु सूर्यं वद ने तत्

...श्य किम्

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अस्तूद् स्कस्थः

(ii) | ठो १ | बृ ५ | ठो १ ठार १ १* बृं १ १* रू १ १*घृ १ १* १ १ १ १ |
     |     |     | १ १ १ १ १२ १ ए १ ६० १ १६५२ १

सिर १ १* कॉ १ १* पार १ १* सू १ १ १ १ १ ४५० १ १२० १ ४५०० १ १६५२००
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पुनान्याम

| तो १ | ३४ | १ ९ २ ३ २ ४ २ २ २ २ २ २ २ |

M 11.

( ४४ वर्षो, ४४ रेक्टो )

……... ……...

नीवी सप्त-शतानां कलमार्जन पक्षणे।।

न्यास स्वापत्र क्रियते

| भाष १ विशेष भाष ५ विशेष १ पर ३२ भाष २ ३६ भाष १ ६ | २ ३ ३ ३ |

भाषण ७००

| व्याय रशि २२३ | आय रशि २६० | एतत् कलमार्जन पक्ष ६१ |

| विशेष २२३ २६० ओर्थोकेंद्र ३६० फल + ... + यह ६१ |

एष व्याये।।
म 12.

(43 recto, 43 verso)

सार्थे द्वारशेषवाल भोजने मयमतमेत्
स सिद्धांग वर्षमिस्रै दिनेषामाणिष्यकस्य तु ।
भाष्यार्थे द्वारशेष वजारां स्थितास्य व ।
एषा यथ समुपत्ती कक्ष कालं श्रृद्धि पषिद्धं ॥
वर्तमान विद्यालय दर्शक शताब्द का प्रतीति

10 2 मार्च 13 फीवर 13 = मार्च 1 1 1 1 2 5 मार्च 2 9 2 9
1 9 1 9 1 9 2 9 1 1 2 9 1 9 [ 1 9 9 9 9 9
2 3 2 3 4 2 3 5 2 3 [ 4 4 4

12 33 मार्च 9 19 मार्था 1200 गुणितात्म
9 9 9 1 360 9
2 3

2 10 500 अर्थज्ञात्वम 360 दिवस...तेन सह... वायु-पिण्ड
9 9 6 127

2952 अधुना वायु पिण्ड | दिव 9 | 1800 दिव 600
686 | 727

अर्थज्ञात्वम 360 दिवस | पुन 2582 | पुन 500 | पुन 2582 | 1
486 | 727 | 486 | 727

अर्थज्ञात्वम 360 दिवस...प्रतिदिन... 1800 एवं सब वि-राशिकेन
240

उदात

M 13.

( 42 verso, 42 recto )

आर्थ गुच्छे लयोद्धा सारं भवति

40 मार्च 160 | 13 | एवां चोदें कृता जाता एकेन
9 | 1 | 1

सारं लयोद्धा भिन्निति | 9 | 4 | 27 | फ० 54 एवां

एकेन लोल चत्वारिष्ठ पद्मि संपूर्णे वध | 9 | 4
पादको लगभग चतुर्विन्यास संसारमय तु कि भवतु "

(i) जाता २४। पढ़ित २४। "२२। अर्धा १३। एकबन्म ५४।।
ए....वैश्विक कारण प्रत्येक मूल्य विधि।।

(ii) अपरं वक्षार्थ। विद्याऽनों दिव 'कि प्रथमें खन्नकेखु यो-

भिलिखित। अपस्य प्रश्ना विधि | २० १ १ | गुणये। गुणिता
| १ १ ३ |

जाता | २० ३ १ | छेँद | २० १ १ | भागे...जातं फलं ह
| १ २ ३ | १ २ १ |

१०।। एष विद्याः दिव""भवति। अत उपरिमाशवन्ध्वकस्य एष
गुणाकारं भवति।।

M 14.
(५० वर्षो, ५० रेक्टो।)

(i) दशमे खुस शतं लवशं अर्धेण लम्बते X कति।।

एक राशिस्तु कलना गाण्तल प्रक्रिया कुः:

| १ दशमे | । । । फलं ५० |
| १०० दुसरा |
| १ |
| २ |

(ii) अपरं उदाहरण। सातं द्वये त्रयसाधं विद्वांत लम्बतेXकति | २ |
| १ |
| १ |
| १ |
| १ |
| १ |
| १ |
रुपम ॥ अर्धनीपरि संघुण्य... वर्ध क्रममेण च।

अर्धन धर्म गुणये मण्डल संघुणे।

भाजये लब्ध पर्यन्त।

............. वर्षिष्ठ पुजन।

सर्वस्याध्य पुजन पौज उपयोगम् भवतु॥

लिखितं छजजक पुजन गणकराज ब्राह्मणेन।

सर्वस्यादेव शास्त्राणां गणितं सूचिनि तिष्ठति।

आद्यावसाने संसारे उत्पत्ति... महत्

परम्परा वृहति तदा कत्वा शिष्येन परमात्मना

....यदद्वब्रमुत्पन्ना गणित सख्य कारणम्।

यथा...
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